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# Unsteady MHD flow in a porous channel with an exponentially decreasing suction

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## Abstract

The unsteady two-dimensional laminar flow of a viscous incompressible and electrically conducting fluid through a channel with the one wall impermeable and the other porous, under the influence of a transverse magnetic field is investigated. The analysis is carried out using the integral method and the expressions for various flow characteristics are obtained and discussed quantitatively.

## 1 Introduction

The study of flow through a channel with permeable walls not only possesses a theoretical appeal but also models biological and engineering systems. Some examples in living organisms are fluid transport mechanisms among which are blood flow in the circulatory system, airflow in the airways, flow system for transporting lymph, urinary circulatory system and transpiration cooling. Another important example is in nuclear power stations where the separation of Uranium $U_{235}$ from $U_{238}$ by gaseous diffusion takes place. In a pioneering work, Berman [1] presented an 'exact' solution of the Navier-Stokes equations that describes the steady two-dimensional flow of an incompressible viscous fluid along a channel with parallel rigid porous walls, the flow being driven by uniform suction or injection at the walls. He assumed that the solution had a similarity form and thereby reduced the problem to that of a non-linear ordinary differential equation of fourth order with a pair of boundary conditions at each wall. Sellars [2] extended Berman's work to high suction Reynolds number. Yuan and Finklestein [3] considered the flow in a porous circular pipe, obtaining solutions for small suction and injection values and an asymptotic solution valid at large injection values. Macey [4] analysed the flow in renal tubules as viscous flow through a circular tube of uniform cross-section and permeable boundaries by prescribing the radial velocity at the walls as an exponentially decreasing function of axial distance. Terrill and Thomas [5] developed a theory further by considering laminar flow in a porous pipe with constant suction or injection applied at the wall. They found that for values of Reynolds number $R$ in the range, $2.3 < R < 9.1$, there were no solutions in similarity form. Therefore, it is helpful to consider how the flows evolve, becoming unstable and bifurcating, as the Reynolds number increases. In this respect, numerous authors (e.g., [6], [7], [8], [9], etc.) have developed and generalised this exact solution.

Over the years, the remarkable influence of electromagnetic force on a conducting fluid has been of great interest to scientists and engineers especially from the practical point of view in understanding the operation of MHD power generators or accelerators and seawater propulsion. Furthermore, it has been established that the biological systems in general are greatly affected by the application of external magnetic field. In the investigations reported in [10], it was observed that the heart pumping rate decreases by exposing the human body to an external magnetic field.

Mathematically speaking, the problem of Magneto-Hydrodynamic (MHD) flow involves solving the basic fluid dynamical equations together with that of electromagnetism, see [11], in order to simulate real life problems. Mori [12] considered the flow between two vertical plates which are electrically non-conducting. He assumed the wall temperature to vary linearly in the direction of the flow and the existence of a heat source in the vertical channel. Without any heat sources, the MHD flow in a vertical parallel plate channel was discussed by Yu [13]. MHD free convection between two parallel plates was studied by [14]. The problem of fully developed flow between two vertical plates taking into account the radiation effects was studied by [15].

MHD oscillatory flow of blood through channels of variable cross-section was investigated by [16]. Recently, [17] investigated the problem of steady MHD variable viscosity plane-Poiseuille flow. The most amazing and significant result in their study is the presence of a turning point in the flow field at low magnetic intensity.

In the present work, the unsteady flow in a channel with a permeable boundary in the presence of an imposed transverse magnetic field is considered. Our objective is to study the combined effects of magnetic field and wall absorption on the flow characteristics. In the following Sections, the problem is formulated, analysed and discussed.

## 2 Mathematical Formulation

We consider the steady flow of a viscous electrically conducting fluid through a long channel, with one wall permeable and the other impermeable, under the influence of an externally applied homogeneous magnetic field. It is assumed that the fluid is incompressible with small electrical conductivity and the electromagnetic force produced is very small. Take a Cartesian coordinate system $(x, y)$ where $Ox$ lies along the center of the channel, $y$ is the distance measured in the normal section. Let $u$ and $v$ be velocity components in the directions of $x$ and $y$ increasing respectively and $p$ the pressure. Then, in two-dimensions, the governing equations of continuity,

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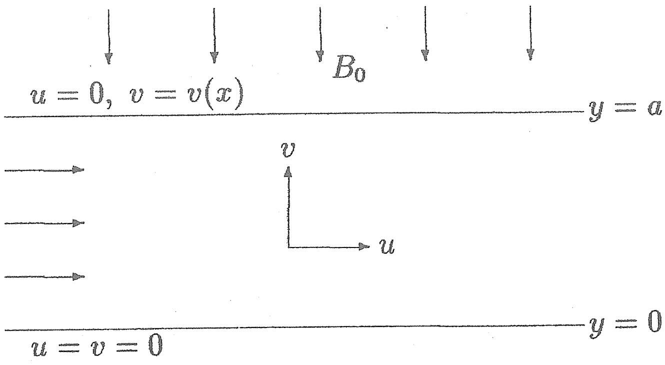


Figure 1: Schematic diagram of the problem

momentum and energy are given in dimensional form as

$$\frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*} = 0, \tag{1}$$

$$\frac{\partial u_*}{\partial t_*} + u_*\frac{\partial u_*}{\partial x_*} + v_*\frac{\partial u_*}{\partial y_*} = -\frac{1}{\rho}\frac{\partial p_*}{\partial x_*} + \frac{\partial u_*^2}{\partial y_*^2} - \frac{\sigma_e B_0^2}{\rho}u_*, \tag{2}$$

where $\mathbf{B} = (0, B_0)$ is the magnetic field vector.

The appropriate boundary conditions are

$$u_* = 0, \quad \text{on} \quad y = a, \tag{3}$$

$$u_* = 0, \quad \text{on} \quad y = 0, \tag{4}$$

where $a$ is the width of the channel, $\sigma_e$ the electrical conductivity, $\rho$ the fluid density, $\nu$ the kinematic viscocity, $c_p$ the specific heat at constant pressure, and $k$ is the thermal conductivity.

The absorption of fluid at the walls is accounted by prescribing the flow flux as an arbitrary function of $x_*$ i.e.

$$\int_0^a u_* dy_* = Uf\left(\frac{x_*}{a}\right)(1 + \varepsilon e^{\frac{int^*\nu}{a^2}}), \tag{5}$$

where $U$ is the initial characteristic flow velocity (i.e. at $x_* = 0$) and $f(x_*/a)$ is the flux function that describes the rate of fluid absorption through the permeable wall such that $f(0) = 1$. It is also very important here to emphasise that $f(x_*/a) =$ constant corresponds to the case of a channel with impermeable boundaries. Introducing the following non-dimensional quantities:

$$u = \frac{u_*}{U},\ v = \frac{v_*}{U},\ x = \frac{x_*}{a},\ y = \frac{y_*}{a},\ p = \frac{p_*}{\rho U^2},$$

$$t = \frac{t_*}{(a^2/\nu)},\ R_e = \frac{Ua}{\nu},\ H = \sqrt{\frac{\sigma_e B_0^2 a}{\rho U}} \tag{6}$$

and substituting into equations (1)-(2), we obtain

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-h(x,t)+\frac{1}{R_e}\frac{\partial^2 u}{\partial y^2}-H^2u. \tag{8}$$

The boundary conditions reduce to

$$u=0 \quad \text{on} \quad y=0 \tag{9}$$

$$u=0 \quad \text{on} \quad y=1 \tag{10}$$

with

$$\int_0^1 u\,dy=f(x)(1+e^{int}) \tag{11}$$

where $R_e$ is the Reynolds, $H$ is the Hartmann number and $h(x,t)=\partial p/\partial x$ is the pressure gradient.

From equation (7) we have

$$v=-\int_0^y \frac{\partial u}{\partial x}dy. \tag{12}$$

Let the solution of the equations (7)-(8) be of the form

$$\begin{aligned} u(x,y,t)&=u_0(x,y)+\varepsilon u_1(x,y)e^{int}\\ v(x,y,t)&=v_0(x,y)+\varepsilon v_1(x,y)e^{int}\\ h(x,t)&=h_0(x)+\varepsilon h_1(x)e^{int} \end{aligned} \tag{13}$$

where $\varepsilon$ is the small amplitude of oscillation and hence we can assume square and higher order terms of $\varepsilon$ to be of negligibly small magnitude, and $n$ is the pulse. Substituting (13) into (7) and (8), and comparing the coefficients of zero and first order terms of $\varepsilon$ on both sides, we obtain:

$$\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}=0, \tag{14}$$

$$u_0\frac{\partial u_0}{\partial x}+v_0\frac{\partial u_0}{\partial y}=-h_0+\frac{1}{R_e}\frac{\partial^2 u_0}{\partial y^2}-H^2u_0, \tag{15}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0, \tag{16}$$

$$u_0\frac{\partial u_1}{\partial x}+u_1\frac{\partial u_0}{\partial x}+v_1\frac{\partial u_0}{\partial y}+v_0\frac{\partial u_1}{\partial y}=\frac{1}{R_e}\frac{\partial^2 u_1}{\partial y^2}-h_1-inu_1-H^2u_1. \tag{17}$$

Initial at $t = 0$,

$$u = u_0 + \varepsilon u_1 \quad (18)$$
$$v = v_0 + \varepsilon v_1 \quad (19)$$
$$h = h_0 + \varepsilon h_1. \quad (20)$$

# 3 Method of Solution

From equations (14) and (16), we have

$$v_0 = -\int_0^y \frac{\partial u_0}{\partial x} dy, \quad (21)$$
$$v_1 = -\int_0^y \frac{\partial u_1}{\partial x} dy, \quad (22)$$

where $y$ is any point between 0 and 1.

Using (21) and (22), equations (15) and (17) become

$$u_0 \frac{\partial u_0}{\partial x} - \frac{\partial u_0}{\partial y} \int_0^y \frac{\partial u_0}{\partial x} dy + h_0 = \frac{1}{Re} \frac{\partial^2 u_0}{\partial y^2} - H^2 u_0, \quad (23)$$
$$u_0 \frac{\partial u_1}{\partial x} + u_1 \frac{\partial u_0}{\partial x} - \frac{\partial u_0}{\partial y} \int_0^y \frac{\partial u_1}{\partial x} dy - \frac{\partial u_1}{\partial y} \int_0^y \frac{\partial u_0}{\partial x} dy + h_1 = \frac{1}{Re} \frac{\partial^2 u_1}{\partial y^2}$$
$$-(H^2 + in) u_1. \quad (24)$$

The additional two equations, which govern the moment of momentum in the $x$ - direction can be obtained by multiplying the equations (23) and (24) throughout by $y$ to get

$$y u_0 \frac{\partial u_0}{\partial x} - y \frac{\partial u_0}{\partial y} \int_0^y \frac{\partial u_0}{\partial x} dy + y h_0 = y \frac{1}{Re} \frac{\partial^2 u_0}{\partial y^2} - y H^2 u_0, \quad (25)$$
$$y u_0 \frac{\partial u_1}{\partial x} + y u_1 \frac{\partial u_0}{\partial x} - y \frac{\partial u_0}{\partial y} \int_0^y \frac{\partial u_1}{\partial x} dy - y \frac{\partial u_1}{\partial y} \int_0^y \frac{\partial u_0}{\partial x} dy + y h_1 = \frac{y}{Re} \frac{\partial^2 u_1}{\partial y^2}$$
$$-y(H^2 + in) u_1. \quad (26)$$

Integrating (by parts) equations (23) to (26) from $y = 0$ to $y = 1$ term by term and simplifying gives

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$$2\int_0^1 u_0\frac{\partial u_0}{\partial x}dy + h_0 = \frac{1}{Re}\left(\frac{\partial u_0}{\partial y}\bigg|_{y=1} - \frac{\partial u_0}{\partial y}\bigg|_{y=0}\right) - H^2\int_0^1 u_0 dy, \tag{27}$$

$$2\int_0^1 u_0\frac{\partial u_1}{\partial x}dy + 2\int_0^1 u_1\frac{\partial u_0}{\partial x}dy + h_1 = \frac{1}{R_e}\left(\frac{\partial u_1}{\partial y}\bigg|_{y=1} - \frac{\partial u_1}{\partial y}\bigg|_{y=0}\right)$$

$$-(H^2 + in)\int_0^1 u_1 dy, \tag{28}$$

$$2\int_0^1 \left(yu_0\frac{\partial u_0}{\partial x}\right)dy + \frac{h_0}{2} = \frac{1}{R_e}\frac{\partial u_0}{\partial y}\bigg|_{y=1} - H^2\int_0^1 yu_0 dy, \tag{29}$$

$$2\int_0^1 yu_0\frac{\partial u_1}{\partial x}dy + 2\int_0^1 yu_1\frac{\partial u_0}{\partial x}dy + \frac{h_1}{2} = \frac{1}{R_e}\frac{\partial u_1}{\partial y}\bigg|_{y=1}$$

$$-(H^2 + in)\int_0^1 yu_1 dy. \tag{30}$$

Since the integral equations (27) to (30) contain six unknowns $u_0, u_1, h_0$ and $h_1$ of which $h_0, h_1$ are functions of $x$ only and while $u_0, u_1$ are functions of both $x$ and $y$. We choose the solutions for $u_0$ and $u_1$ in polynomial form as,

$$u_0 = f_0(x) + yf_1(x) + y^2 f_2(x) + y^3 g(x), \tag{31}$$

$$u_1 = F_0(x) + yF_1(x) + y^2 F_2(x) + y^3 G(x), \tag{32}$$

where $f_0, f_1, f_2, g, F_0, F_1, F_2, G$, are arbitrary functions of $x$. These unknowns can be obtained by substituting the conditions (15) and (17) into the equations (31) and (32), to get

$$f_0 = F_0 = 0, f_1 = 6f + \frac{1}{2}g,\ f_2 = -\left(6f + \frac{3}{2}g\right), \tag{33}$$

$$F_1 = 6f + \frac{1}{2}G,\ F_2 = -\left(6f + \frac{3}{2}G\right).$$

Thus the assumed polynomial solutions in terms of unknowns i.e. $g(x), G(x), f$ become:

$$u_0 = 6f(y - y^2) + (y^3 - 3/2y^2 + y/2)g, \tag{34}$$

$$u_1 = 6f(y - y^2) + (y^3 - 3/2y^2 + y/2)G, \tag{35}$$

Substituting equations (34) to (35) into the equations (27)-(30), we obtain

$$g\frac{dg}{dx} - \frac{420}{R_e}g = -420\left(\frac{12f}{5}\frac{df}{dx} + H^2 f + h_0\right), \tag{36}$$

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$$(g-12f)\frac{dg}{dx}-g\left(12\frac{df}{dx}+7H^2+\frac{1}{2R_e}\right)=-420f\left(\frac{12}{5}\frac{df}{dx}+\frac{12}{R_e}+H^2+h_0\right)(37)$$

$$g\frac{dG}{dx}+G\left(\frac{dg}{dx}-\frac{420}{R_e}\right)=-420\left(\frac{24f}{5}\frac{df}{dx}+\{H^2+in\}f+h_1\right), \quad (38)$$

$$(g-12f)\frac{dG}{dx}+G\left(\frac{dG}{dx}-12\frac{df}{dx}-7(H^2+in)-\frac{420}{R_e}\right)=$$
$$-420f\left(\frac{24}{5}\frac{df}{dx}+\frac{24}{R_e}+H^2+in\right)+420\left(\frac{g}{35}-h_1\right). \quad (39)$$

We eliminate $h_0$ and $h_1$ from equations (36) to (39) and obtain;

$$\frac{dg}{dx}+\frac{1}{f}\left(\frac{df}{dx}+\frac{35}{R_e}+\frac{7H^2}{12}\right)g=0, \quad (40)$$

$$\frac{dG}{dx}+\frac{1}{f}\left(\frac{df}{dx}+\frac{35}{R_e}+\frac{7}{12}(H^2+in)\right)G=-\frac{g}{f}\frac{df}{dx}-\frac{dg}{dx}. \quad (41)$$

Equations (40) and (41) are solved using $f=e^{-x}$ and the results. The solutions are

$$g = a_0\exp\left(\frac{12xR_e-420e^x-7e^xH^2R_e}{12R_e}\right), \quad (42)$$

$$G = -i\left[(R_eH^2+60)a_0\exp\left(\frac{12xR_e-420e^x-7e^xH^2R_e}{12R_e}\right)\right.$$
$$\left.+inR_ea_1\exp\left(\frac{12xR_e-420e^x-7e^xH^2R_e-7ie^xnR_e}{12R_e}\right)\right]/(nR_e), \quad (43)$$

$$h_0 = \frac{1}{5040}(-60480e^{-x}-5040H^2e^{-x}R_e+a_0^2B_4^2\{420e^x+7e^xH^2R_e-12R_e\}$$
$$+12096e^{(-x)^2}R_e)/R_e, \quad (44)$$

$$h_1 = -\frac{1}{5040}(60480e^{-x}R_enH^2+5040iR_e^2n^2e^{-x}R_e$$
$$+B_5^2(50400ia_0^2e^x+1680ia_0^2e^xH^2R_e-1440ia_0^2R_e-24ia_0^2R_e^2H^2+14ia_0^2R_e^2H^4e^x)$$
$$+B_5B_5a_0a_1(24nR_e^2-840nR_ee^x-14nR_e^2e^xH^2-7in^2R_e^2e^x)$$
$$-24192e^{(-x)^2}R_e^2n)/(R_e^2n) \quad (45)$$

We then take the real part of equation (13) in order to analyse the solutions.

$$u = \left(\left(6e^{-x}(y-y^2)+a_1B_4\cos\left(\frac{7}{12}e^xn\right)\left(\frac{1}{2}y-\frac{3}{2}y^2+y^3\right)\right)\cos(nt)\right.$$
$$\left.-\frac{\left(-60a_0B-R_eH^2a_0-a_1B_4\cos\left(\frac{7}{12}e^xn\right)nR_e\right)\left(\frac{1}{2}y-\frac{3}{2}y^2+y^3\right)\sin(nt)}{nR_e}\right)$$
$$\varepsilon+6e^{-x}(y-y^2)+a_0B_4\left(\frac{1}{2}y-\frac{3}{2}y^2+y^3\right) \quad (46)$$

*

$$
\begin{aligned}
v \;=\;& \Big(-\frac{1}{12}a_1(12R_eB_4B_5 - 420B_2B_5 - 7H^2R_eB_2B_5 - 7nR_eB_2B_3)\\
&\left(\frac{1}{4}y^2 - \frac{1}{2}y^3 + \frac{1}{4}y^4\right)\cos(nt)/R_e - \left(\frac{5a_0(12R_e - 420e^x - 7e^xH^2R_e)B_4}{R_e}\right)\\
&\left(R_e - 35e^x - \frac{7}{12}e^xH^2R_e\right)H^2a_0B_4\\
&-\frac{1}{12}a_1n(-12R_eB_4B_3 + 420B_2B_3 + 7H^2R_eB_2B_3 - 7nR_eB_3B_5)\Big)\\
&\left(\frac{1}{4}y^2 - \frac{1}{2}y^3 + \frac{1}{4}y^4\right)\varepsilon\\
&+\frac{a_0(12R_e - 420e^x - 7e^xH^2R_e)B_4\left(\frac{1}{4}y^2 - \frac{1}{2}y^3 + \frac{1}{4}y^4\right)}{12R_e} \qquad (47)
\end{aligned}
$$

where

$$
\begin{aligned}
B_0 &= a_0\exp\left(\frac{12xR_e - 420e^x - 7e^xH^2R_e - 7ie^xnR_e}{12R_e}\right),\\
B_1 &= a_0\exp\left(\frac{12xR_e - 420e^x - 7e^xH^2R_e - 7ie^xnR_e}{12R_e}\right),\\
B_2 &= a_0\exp\left(\frac{2xR_e - 35e^x - \frac{7}{12}e^xH^2R_e}{R_e}\right)\\
B_3 &= \sin\left(\frac{7}{12}e^xn\right),\\
B_4 &= a_0\exp\left(\frac{xR_e - 35e^x - \frac{7}{12}e^xH^2R_e}{R_e}\right)\\
B_5 &= \cos\left(\frac{7}{12}e^xn\right),
\end{aligned}
$$

and where $a_0, a_1$ are undetermined constants The wall shear stress is given as

$$
\begin{aligned}
\tau_w &= \mu\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\\
&= -\frac{1}{2}\Big(12\cos(nt)R_ene^{-x} + \cos(nt)R_ea_1\cos\left(\frac{7}{12}e^xn\right)B_6\\
&\quad +60\sin(nt)a_0B_6 + \sin(nt)R_eH^2a_0B_6 + \sin(nt)R_ea_1n\sin\left(\frac{7}{12}e^xn\right)B_6\Big)/(R_en) -\\
&\quad -\frac{12e^{-x}R_en = a_0R_enB_6}{R_en} \qquad \text{at } y = 0, 1, \qquad (48)
\end{aligned}
$$

where

$$
B_6 = -\frac{12xR_e - 420e^x - 7e^xH^2R_e}{12R_e}.
$$

# 4 Graphical Results and Discussion

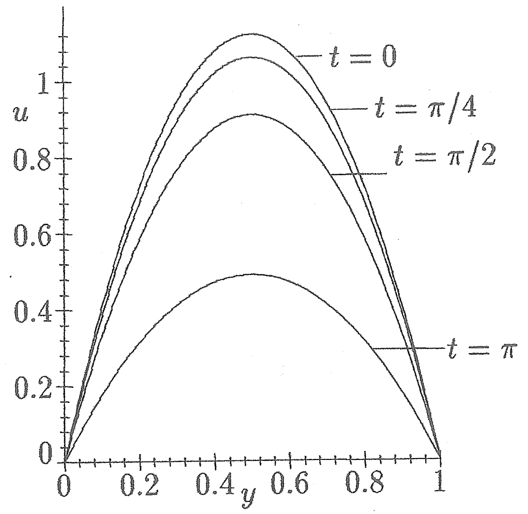


Figure 2: Axial velocity profile, $H = 5, R_e = 5, n = 1/2, a_0 = a_1 = 1, \varepsilon = 1, x = 1$

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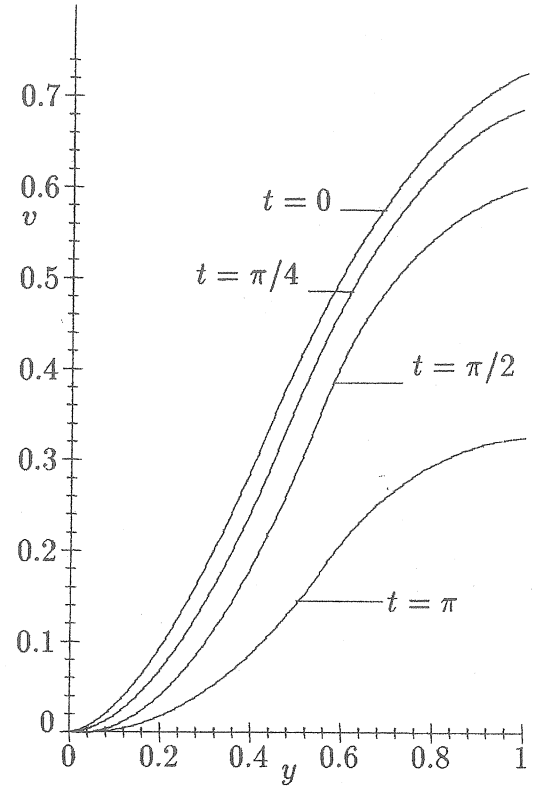


Figure 3: Normal velocity profile, $H = 5, R_e = 5, n = 1/2, a_0 = a_1 = 1, \varepsilon = 1, x = 1$

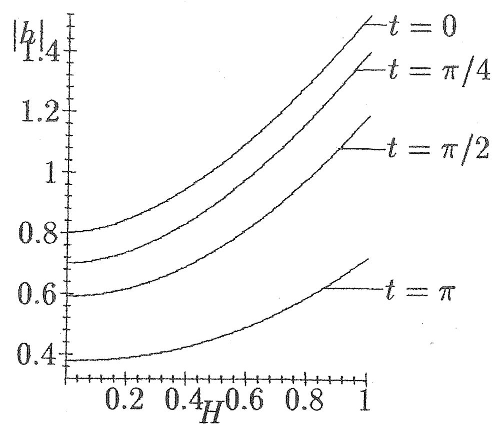


Figure 4: Variation of pressure gradient with the magnetic field

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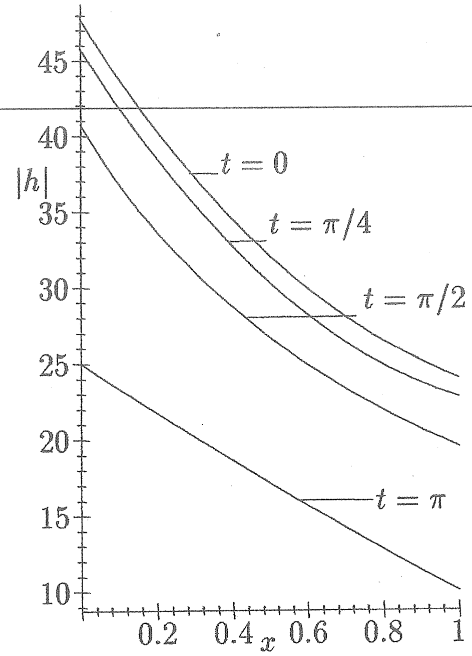


Figure 5: Variation of pressure gradient with axial distance $H = 5, R_e = 5, n = 1/2, a_0 = a_1 = 1$

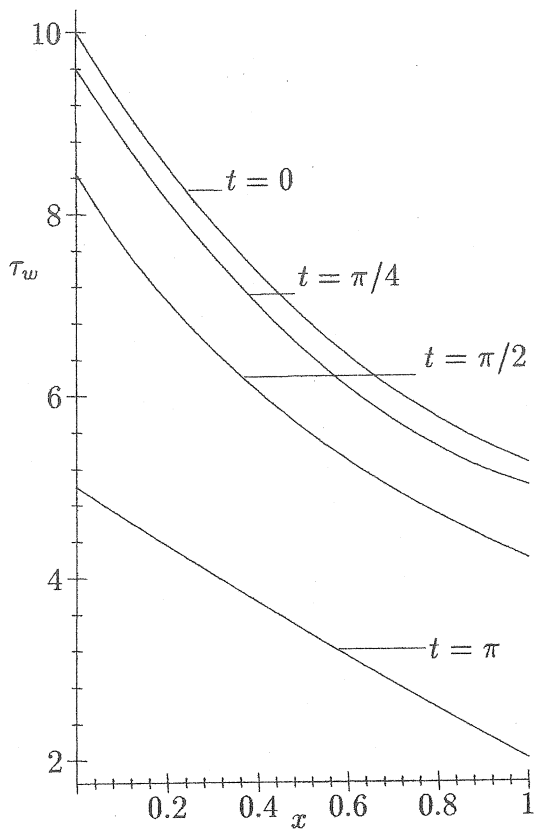


Figure 6: Variation of wall shear stress with axial distance $H = 5, R_e = 5, n = 1/2, y = 1, a_0 = a_1 = 1$

* For numerical discussion of the above analytical results, we have taken $a_0 = a_1 = 1$, $\varepsilon = 1$, $H = 0, 1, 2, 5$, $R_e = 5$, $t = 0, \pi/4, \pi/2, \pi$.

Figure (2) shows the axial velocity profile which is parabolic in nature and fluctuate with maximum magnitude at $t = 0$, and minimum at $t = \pi$ due to the unsteadiness effect. The effect of exponentially decreasing wall suction is shown in Figure (3). The normal velocity is maximum at the wall and also fluctuate due to the unsteadiness effect. From the mathematical point of view, we observed that the flow velocity is reduced due to an increase in Magnetic field intensity. This ultimately leads to an increase in magneticity of the fluid pressure gradient as shown in Figure (4). The fluid pressure gradient decreases with axial distance due the suction effect (Figure 5).

The wall shear stress decreases with an increase in axial distance (Figure 6). This is clearly due to the exponentially decreasing suction effect, however, the effect of the magnetic field is to increase the wall shear stress.

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