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BOOK OF ABSTRACTS

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1 Keynote Presentations

SAMSA Masamu Advanced Study Institute and the Collaborative Research Network

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Auburn University

Abstract

The Collaborative Research Network (CRN) is a five-year project consisting of research faculty from Sub-Saharan Africa, US, Canada, and Europe forming four research teams in Pure Mathematics and three research teams in Applied Mathematics. The primary objective of the network, as currently funded by the US National Science Foundation (NSF), is to drastically increase the research collaboration and the quantity and quality of new PhDs from diverse backgrounds in mathematical sciences in US and Sub-Saharan Africa. To achieve this objective, CRN will synergistically utilize the SAMSA Annual Conference, Masamu Advanced Study Institute (MASI) and Research Workshops, Simons Foundation Program, and year-long research activities. In this presentation, we will discuss the CRN structure, goals, and vision.

Model Uncertainty and Robust Duality in Finance

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Abstract

A celebrated financial application of convex duality theory gives an explicit relation between the following two quantities:

- (i) The optimal terminal wealth $X^*(T) := X_{\phi^*}(T)$ of the classical problem to maximise the expected U-utility of the terminal wealth $X_{\phi}(T)$ generated by admissible portfolios $\phi(t); 0 \leq t \leq T$ in a market with the risky asset price process modelled as a semimartingale
- (ii) The optimal scenario dQ^*/dP of the dual problem to minimise the expected V-value of dQ/dP over a family of equivalent local martingale measures Q . Here V is the convex dual function of the concave function U.

1) In the first part of this talk we consider markets modelled by It-Lvy processes, and we present a new approach to the above result in this setting, based on the maximum principle in stochastic control theory. An advantage with our approach is that it also gives an explicit relation between the optimal portfolio ϕ^* and the optimal scenario Q^* , in terms of backward stochastic differential equations. This can be used to obtain a general formula for the optimal portfolio $\phi^*(t)$ by means of the Malliavin derivative.

2) In the second part we extend our study to a robust portfolio problem and its dual. More specifically, we study the portfolio problem and its dual under model uncertainty, and we prove a corresponding duality equivalence in that setting. Our approach allows us to obtain explicit relations between the solutions of the robust primal and the robust dual problem.

We illustrate the results with explicit examples.

The presentation is based on recent joint work with Agns Sulem, INRIA-Rocquencourt, France.

Total Variation Based Regularization in Image Restoration: Fast Solvers, Spatially Adaptive Regularization and Sparsity Enhancing Quasi-Norm Regularization

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Abstract

Total variation regularization has been used successfully over the past years in the restoration of digital images, which are degraded by convolution operations and by additive noise (Gaussian or impulse-type). A key feature is to find the right balance between data fidelity and filtering of noise by regularization. This adjustment is achieved by selecting a regularization parameter. While most available approaches rely on a given regularization parameter, some try to identify this scalar automatically in order to reduce user intervention.

In this talk we discuss a novel approach which relies on a spatially (over the image domain) distributed regularization parameter. This parameter is selected fully automatically based on a confidence interval technique depending on the distribution of the maximum of a number of random variables. Besides analytical justification of the proposed technique, numerical results are provided to assess its practical performance.

While it is known that the total variation regularization aims at gradient sparse images through minimizing the L1-norm of an associated measure. Often, minimizing an objective with an L1-regularization results from considering the convex relaxation closest to the 0-norm regularization, i.e., minimizing the support set of a quantity of interest (like the gradient in the aforementioned application) in the reconstruction process.

In a second part of the talk, non-convex quasi-norm regularizations are considered in order to reduce the compromise between the NP-hard minimization of the 0-norm and the associated convex relaxation. An efficient numerical solver is introduced and its practical performance in several application fields, including image processing, sparse optimal control and machine learning, is assessed.

Three Gems from the Arithmetic of Fields

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Abstract

In this talk we present three beautiful problems from the arithmetic of fields. The first is classical involving factorization and class numbers, the second involves algebraic geometric codes and interaction with other fields and the third curves and their automorphisms in characteristic p and 0 . Each of these problems has led to interesting new work, open questions and conjectures, which will be discussed in the talk.

Robust Estimation of Regression Model Parameters with Applications

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Abstract

Robust estimation of linear model parameters has been studied since the 1970's; however, the literature on the robust estimation of the parameters of general nonlinear regression models is quite sparse. Such models have useful applications and include generalized linear models, signal processing models, and nonlinear ODEs as special cases. We will, in particular, consider estimators obtained via minimization of rank-based dispersion functions. It turns out that such estimators are efficient relative to classical estimators. They are also robust to outliers in response space and, with some modification, to outliers in design space. We will consider some real and simulated examples.

MHD Nanofluid Bioconvection due to Gyrotactic Microorganisms of over a Permeable Vertical Plate

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Abstract

In this paper, a similarity solution for hydromagnetic nanofluid bioconvection due to gyrotactic microorganisms past a permeable vertical moving surface is presented. Nanofluid bioconvection is generated by the combined effects of buoyancy forces and magnetic field on the interaction of motile microorganisms and nanoparticles. The model employed for the nanofluid transport equations incorporates the effects of Brownian motion and thermophoresis. The nonlinear governing partial differential equations are first transformed into a set of ordinary differential equation, and then tackled numerically using shooting technique with a fourth-order Runge-Kutta-Fehlberg integration scheme. A parametric study of the entire flow regime is carried out to illustrate the effects of the pertinent parameters, viz., bioconvection Lewis number Lb , traditional Lewis number Le , bioconvection Peclet number Pe , buoyancy ratio parameter Nr , bioconvection Rayleigh number Rb , Brownian motion parameter Nb , thermophoresis parameter Nt , Hartmann number Ha , Grashof number Gr , Eckert number Ec , the microorganisms concentration difference parameter Ω and the suction/injection parameter f_w on the velocity, temperature, nanoparticles volume fraction and motile microorganisms density profiles as well as the skin friction coefficient, the local Nusselt number, the local Sherwood number and the local density number of the motile microorganisms.

Keywords: Hydromagnetic; Nanofluid; Permeable plate; Gyrotactic microorganisms; Bioconvection.

The directed span of a T_0 -quasi-metric space

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Abstract

Isbell [6] constructed the hyperconvex (or injective) hull of a metric space. Later his theory was rediscovered independently several times, for instance by Dress [3] in his theory of the tight span of a metric space. Lawvere [9] had observed that metrics that do not necessarily satisfy the symmetry condition (they will be called *quasi-metrics* in the following) can be understood as quantified partial orders. This crucial observation indeed helps explain many similarities (compare e.g. [8]) that exist in the classical theories of metric spaces and the theory of partially ordered sets, since in fact both theories can be understood as special cases of the more general theory of T_0 -quasi-metric spaces. In the last years many aspects of the theory of analysis in metric spaces have been generalized to quasi-metric spaces (see for instance [2,10,11]).

In our talk we shall consider the injective hull in the category of T_0 -quasi-metric spaces (compare for instance [4,5,7,12]). Generalizing Isbell's theory of endpoints in metric spaces, we shall in particular discuss the concept of an endpoint in the quasi-metric theory (see for instance [1]). A simple example will illustrate how the injective hull in the category of T_0 -quasi-metric spaces generalizes the better known Dedekind-MacNeille completion for partially ordered sets.

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A stochastic model for in-host malaria parasite infection of red blood cells

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Abstract

In this study, we consider how the stability of the basic malaria model is altered by the Brownian diffusion matrix. First, we consider a model with constant diffusion matrix and show that for this diffusion structure, the revised Anderson and May model possesses two steady states the parasite-free and parasite-present steady states whose stability is degraded by the diffusion term. This type of diffusion can be used to study infections whose states can switch from parasite-free to parasite-present and vice versa, such as plasmodium falciparum. Secondly, we consider a model with linear diffusion matrix and show that the model possesses only a parasite-present steady state. This model can be used to study infections which maintain unstable parasite-present states with relapse tendencies, like plasmodium vivax.

Some applications of graph theory in knot theory

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Abstract

Knots (links) are fascinating geometrical objects. At the same time they are very simple to visualise and yet remarkably hard to analyse. In mathematics, we look at knotted and unknotted loops of string and analyse the forms and relationships between them. Knots (links) can be analysed using different tools in topology, group theory, algebra and graph theory. In addition, there is a new branch called combinatorial knot theory.

There are several research problems in knot (link) theory which continue to generate a lot of interest and activity. One problem which is still of interest is experimenting with transferring techniques in graph theory into knot theory. It has been shown in the literature that each signed planar graph has a corresponding link diagram and vice-versa. Thus each planar graph which is not signed has a corresponding link universe. This provides a bridge between knot theory and graph theory. There are many notions of graphs which have been transferred successfully into knots (links), one example is graph polynomials.

In this talk we address two questions which we can easily translate from graphs to links. First, we use a planar graph to count the number of components of the corresponding link by defining an algorithm on a graph. Finally, we compare the path-width of a planar graph and the bridge number of the corresponding link.

Keywords: knot; graph; component number; bridge number.

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Finite Element Approximations in Solid and Fluid Mechanics

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Abstract

The finite element method is a powerful approach for the development of approximate solutions to boundary-value and initial-boundary-value problems, characterized typically by systems of partial differential equations. The method has a sound mathematical basis, and is extensively researched and used in the natural sciences and engineering. The first objective is to present an overview of the finite element method and the convergence theory for linear elliptic problems. Secondly, examples are presented of applications and extensions to a range of nonlinear problems in solid and fluid mechanics. The focus in the applications is on the interplay between numerical analysis, development of algorithms, and computational implementation.

2 ALG: Abstract Algebra & Group Theory

Counting Irreducible Polynomials of Degree r over \mathbb{F}_{q^n} and Generating Goppa Codes Using The Lattice of Subfields of $\mathbb{F}_{q^{nr}}$

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Abstract

The problem of finding the number of monic irreducible polynomials of degree r over \mathbb{F}_{q^n} is considered in this paper. By considering the fact that an irreducible polynomial of degree r over \mathbb{F}_{q^n} does not have a root in a subfield \mathbb{F}_{q^s} of $\mathbb{F}_{q^{nr}}$ if and only if $(\frac{nr}{s}, r) \neq 1$, we show that Gauss' formula for the number of monic irreducible polynomials can be derived by merely considering the lattice of subfields of $\mathbb{F}_{q^{nr}}$. We also use the lattice of subfields of $\mathbb{F}_{q^{nr}}$ to discuss the conditions under which it is possible to generate a Goppa code using a root lying in a subfield of the splitting field of the Goppa polynomial.

Irreducible elements in the ring of integers modulo n

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Abstract

In this paper we determine all irreducible elements in the ring of integers modulo n . If $n = p_1 p_2 \cdots p_k$ and the p_i 's are distinct primes we show that \mathbb{Z}_n has no irreducible elements. We find irreducible elements in \mathbb{Z}_n where $n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}$ and the p_i 's are distinct primes with at least one prime having power $e_i > 1$. All p_i 's with $e_i > 1$ generate the whole set of irreducible elements in \mathbb{Z}_n . The total number of irreducible elements in \mathbb{Z}_n is $\sum_i \frac{\phi(n)}{p_i}$ where p_i is a prime with $e_i > 1$ and $\phi(n)$ is the number of units in \mathbb{Z}_n .

Keyword: Irreducible elements.

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A note on order one cyclotomic polynomials

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Abstract

It is a well known fact that, if p is an odd prime, then the p^{th} - elementary cyclotomic polynomial $\Phi_p(x)$ has an associated p - Eisenstein polynomial $\widehat{\Phi}_p(x)$. In this talk, we will extend this construction and show that, every order one elementary cyclotomic polynomial $\Phi_{2^s p^t}(x)$ has an associated p - Eisenstein polynomial $\widehat{\Phi}_{2^s p^t}(x)$. In addition, for each $\Phi_{2^s p^t}(x)$, we investigate the divisibility (with respect to the prime p) of the coefficients of $\widehat{\Phi}_{2^s p^t}(x)$. We will also establish analogous results for order one Carlitz cyclotomic polynomials over $\mathbb{F}_q[T]$.

Keywords: Eisenstein polynomials, elementary and Carlitz cyclotomic polynomials.

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ON THE SPLIT EXTENSION $2^9:L_3(4)$ OF THE UNITARY GROUP $U_6(2)$

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Abstract

The Unitary group $U_6(2)$ is a simple group of order $9196830720 = 2^{15} \cdot 3^6 \cdot 5 \cdot 7 \cdot 11$. The group has 16 conjugacy classes of maximal subgroups as listed in the Atlas of Finite Groups [3]. The split extension $2^9:L_3(4)$ is the third largest maximal subgroup of $U_6(2)$ of index 891. In this paper, we use the techniques of coset analysis and the Fischer-Clifford matrices (see [1],[4]) to compute the conjugacy classes and the character table of $2^9:L_3(4)$, respectively. Also, the technique of set intersections [4] will be used to determine the fusion of $2^9:L_3(4)$ into $U_6(2)$. Most of our computations were done in the computer system Magma [2].

Keywords: coset analysis; inertia factor group; fusion map; Fischer-Clifford matrices.

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The Solution to the 3-Variable Frobenius Number Problem

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Abstract

Given a set of relatively prime positive integers a_1, a_2, \dots, a_n , after some point all positive integers are representable as a linear combination of the set with nonnegative coefficients. The last integer that is not so representable is the Frobenius Number, and the problem of finding that number is called the Frobenius problem or coin problem. While the two-variable solution is easy and widely known, and the general solution is NP-hard, there have been several algorithmic solutions of the three-variable problem. Here we discuss a formulaic solution for the Frobenius problem of all relatively prime triples, and a graphic representation of such.

ELEMENTS OF ALGEBRAIC GEOMETRY AND GROEBNER BASES

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Abstract

The purpose of this paper is to solve an algebraic equation and to determine whether it has a set of finite or infinite number of solutions .

Keywords: Polynomial of several variables; Ideal; Groebner bases; Buchbergers algorithm.

References

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CHARACTERIZING RING OF ENDOMORPHISMS AND ITS CENTRE OF FINITE RANK COMPLETELY DECOMPOSABLE TORSION FREE ABELIAN GROUPS.

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Abstract

This paper assesses decompositions of the class of finite rank completely decomposable torsion free Abelian groups into direct sums of their subgroups, the uniqueness of such decompositions up to isomorphism and their application in characterizing their ring of endomorphisms and its centre. To describe the centre of their ring of endomorphisms, we introduce an equivalence relation between the types of fully invariant direct summands of rank one groups and depending on the relation of their types, characterize the centre.

Keywords: Abelian group; types ; fully invariant; centre; endomorphisms.

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On the constant reduction of valued function field in one variable and its automorphism groups

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Abstract

The aim of the talk is to investigate properties of a natural homomorphism between the automorphism group of a function field in one variable over an algebraically closed field and its reductions with respect to special valuations. The general area is known as the Deuring's constant reductions theory. This gives an algebraic way to describe curves over valuation rings, their divisors and their specializations algebraically.

Ergodic Random Fields Of Confidence

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Work-in-progress

Abstract

We consider a set of heterogeneous decision makers (DMs) who have prior beliefs about the price of an object. We introduce a behavioural kernel operator for deviations between DMs objective and subjective probability measures, and construct a dynamical system of (DMs) states of confidence in the object's price. We show how the operator generates a sequence of von Neumann Morgenstern utility functions consistent with the popular Becker-DeGroot-Marschak willingness to pay (WTP) and willingness to accept (WTA) auction mechanism in EXPERIMENTAL ECONOMICS. So in principle, the operator may be testable in the lab. Eigenfunctions from asymptotic expansion of the kernel constitute DMs states of confidence indexed by a convex set of their prior beliefs. Feller semigroup transformation of the eigenfunctions generates ergodic random fields of confidence. Concordant sample paths for the random field mimic pessimists supply (WTA) and optimists demand (WTP) in a market for the object. We provide a closed form expression for probability estimates of large deviation between aggregate supply and demand for the object. We show how to construct a deterministic confidence field; provide a simple example of how probability estimates for large deviations of supply and demand can be used to characterize Minsky moments for the object, i.e., market bubbles and crashes driven solely by states of confidence, that occur almost surely.

Keywords. confidence, market instability, chaos, dynamical systems, ergodic theory, Minsky moments

A Hill Cipher Based on the Kish-Sethuraman (KS) Protocol

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Abstract

In the idealized Kish-Sethuraman (KS) protocol, messages are sent between Alice and Bob each using a secret personal key. This protocol is said to be perfectly secure because both Bob and Alice keep their keys undisclosed so that at all times the message is encrypted by at least one key, thus no information is leaked or shared. In this paper, we propose a realization of the KS protocol through the use of the Hill cipher.

THE CHARACTER TABLE OF THE MAXIMAL SUBGROUP $2^6:(3 \cdot S_6)$ in M_{24}

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Abstract

The largest Mathieu sporadic simple group M_{24} has 9 maximal subgroups as listed in the Atlas of Finite Groups [2]. The split extension group $2^6:(3 \cdot S_6)$ of order 138240 is a maximal subgroup of M_{24} of index 1771. In this paper, we compute the conjugacy classes and the character table of $2^6:(3 \cdot S_6)$ using the techniques of coset analysis and Fischer- Clifford matrices (see [4] and [3]), respectively. Most of our computations were carried out with the aid of the computer system Magma [1].

Keywords: coset analysis; inertia factor group; fusion map; Fischer-Clifford matrices.

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Frobenius number of n variables of a certain form

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Abstract

The classical Frobenius Number Problem has been studied for over a century. Alfred Brauer found an explicit equation for the three variable case for a certain form of third variable. In this paper we proceed to extend his equation to account for n variables of his proposed form. By examining lists of integers which fit this form, as well as looking at the structure of a graph extracted from these lists, we were able to find an algorithm to solve the n variable case. We give an explicit equation for the four variable case and a purely algorithmic way for the arbitrary finite case.

3 BIO: Bio-mathematics & Bio-statistics

Dynamics of the oxygen, carbon dioxide and water interaction across the insect spiracle

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Abstract

All insects exchange gas across their spiracle opening. We explore the dynamics of respiratory gases interactions which are accompanied by the loss of water through insect's spiracles. Here we investigate this interaction by deriving the oxygen, carbon dioxide and water vapor system and analyze if the system parameters have general interpretations. The analysis is carried out in continuous time. The purpose of the research is to determine bounds for the gas volumes and to discuss the complexity and stability of the equilibria. Numerical simulations also demonstrate the dynamics of our model utilizing the new conditions for stability and instability.

Keywords: Discontinuous gas exchange; Respiration; Dessication; Ratio dependence; Complexity and stability.

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A shared component latent variable model for modelling spatial variation in health promotion with applications from Namibia

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Abstract

Background: Gender variation in the spatial pattern of HIV testing and condom use in Namibia was undertaken using Bayesian hierarchical spatial mapping techniques based on data from Namibia Demographic and Health Survey (NDHS) carried out between 2006 and 2007. The main objective of the study is to examine gender-specific spatial variation in HIV testing and condom use in Namibia for targeted health promotion interventions. *Methods:* A number of basic Bayesian structured additive regression (STAR) models were fitted and then followed by shared latent variable component models. Firstly, we modeled HIV testing and condom use in males and females with fixed effects, whereas spatial references to the communities were modeled as structured and unstructured spatial effects. In the second modeling approach, HIV testing and condom use in males and females were modeled jointly with a shared component for random effects using a bivariate conditional autoregression model for the specific-components, and a local co-regionalized Gaussian component for the shared part of the model. By adopting a Bayesian inferential approach, the following priors were assumed: diffused priors for fixed effects, exchangeable Normal priors for unstructured spatial random effects and conditional autoregressive priors for the structured spatial random effects. *Results:* Common and divergent patterns of HIV testing and condom use emerged. Common areas among men and women on HIV testing and condom use were observed in Khomas, Oshikoto, and Oshana, while divergent Caprivi and Karas. Urban influence was also captured in the model. *Conclusions:* The study underscore the usefulness of Bayesian hierarchical mapping model in highlighting areas lagging behind in the uptake of HIV testing and condom use with emphasis on differences between men and women in the same area. The information will be valuable for guiding public health actions that are targeted at the overall reduction of risk-sexual behaviors through HIV testing and the use of condoms.

Keywords: spatial statistical models, hierarchical Bayesian inference, co-regionalized Gaussian prior, shared component latent variable.

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A metapopulation model for cholera transmission dynamics between communities linked by migration

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Abstract

A metapopulation model is developed to describe the spread of cholera between two communities connected by migratory movement. Disease threshold ratios specific to the communities are given, considering a case when the communities are isolated and when the communities are connected. The connection of the threshold ratios to disease spread and stability is discussed. The disease free equilibrium is globally stable whenever, the community corresponding disease threshold ratios are less than one and unstable otherwise. Community specific endemic equilibrium points are unique, locally asymptotically stable, and only exist when the corresponding disease thresholds are greater than unit. Disease spread is explosive in nature at the beginning of the outbreak but more severe in a community with poor facilities relative to a community with more improved facilities. In isolated communities, in the case of endemic cholera, the infection is characterised by a big outbreak, followed by a small episode of the infection. Only one typically big outbreak is observed in the community with improved facilities with no recurrence of the epidemic. In connected communities, movement of individuals across communities not only influences persistence of the infection but also results in a bigger outbreak in a relatively well facilitated community in the long term. Synchronous fluctuation of the population is observed when there is unrestricted movement of both immunologically naive and infected individuals across the communities. Our results suggest that during times of cholera, movement to and from cholera endemic areas should be avoided if the outbreak is to be easily contained.

Keywords: Cholera; Metapopulation; Migration; Disease threshold; synchrony.

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Co-infection of HIV-HCV: A theoretical Perspective

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Abstract

Human Immunodeficiency virus (HIV) accounts for 40-50 million infections globally, while Hepatitis C virus (HCV) an important cause of chronic liver disease, accounts for 150 million infections. The co-infection of HIV and HCV accounts for 8-10 million, making HIV-HCV co-infection a major public health problem. HIV infection accelerates liver disease progression in patients co-infected with Hepatitis C virus, HCV on the other hand, weakens the liver system thus compromising HIV treatment effectiveness.

This paper formulates and analyzes a theoretical model of HIV-HCV co-infection to obtain insight on the transmission dynamics of the two diseases. We determine the basic reproduction number R_0 , of the model and use it to characterize the existence and stability the model equilibria. Simple numerical simulations are carried out to illustrate analytical results derived in the study. The results of the study provide a platform for further studies on the transmission dynamics, control and management of the joint HIV and HCV epidemics.

Socio-economic determinants of infant mortality in Namibia, a case of the 2000 and 2006/7 Demographic and Health surveys (DHS)

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Abstract

This study aimed at establishing whether there is a significant shift in the socio-economic determinants of infant mortality in Namibia between the 2000 and 2006/7 NDHS. Secondary data sets from the two NDHS were separately analysed using descriptive statistics and GLM models were fitted to determine significant determinants of infant mortality for each NDHS. The resulting model specific effects were estimated through the multivariate logistic regression. The result shows an increase in the proportion of infant mortality in 2006/7 NDHS, which is particularly more prominent in the male infants. In addition, delivery of babies at government hospitals and in rural areas continues to be a challenge as this places have experienced an increase in infant mortality in the 2006/7 survey. The outcome of the model fitting indicates a significant shift in the determinants of infant mortality, with the most important determinant of infant mortality in Namibia being the birth order number, followed by the sex of the child whereas the least determinant is the current age of the respondents.

Keywords: Infant mortality; socio-economic determinants; NDHS; Namibia.

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Mathematical Model for Langerhans cells and HIV evolution

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Abstract

Langerhans cells have been shown to be the first target for HIV infection during early HIV infection. We used a mathematical model the effects of incorporating the role of langerhans cells combined with the CD4⁺ T cells on the prognosis of HIV infection. Preliminary results showed that the combined infection increases the production of the free virus making the free virus the dominant source of infection. The growth of the virus was shown to be affected by processes such as the burst size of the infected Langerhans cells and infected CD4⁺ T cells and the time it takes to have secondary infected cells. The contribution of Langerhans cells in early HIV infection may be an important factor to consider to enhance the understanding of the dynamics of HIV infection.

Cost and impact of ART scale up in the presence of treatment failure and loss to follow up: a modeling perspective

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Abstract

There are encouraging signs of scale up of antiretroviral therapy (ART). HIV testing rates have recently increased in many countries including South Africa and other countries with high HIV burden. Further, HIV treatment initiation guidelines have also improved, which resulted for significant increase in the number of individuals receiving treatment. In this study; we constructed a mathematical model for HIV transmission and treatment, structured by time since infection and time since treatment initiation, and fitted it to data from South Africa. As the South African treatment programme matures, the number of patients who require switching to the second line treatment due to first line treatment failure, will increase. Depending on the ratio of the cost of providing second line versus first line HIV treatment, the cumulative cost of the ART programme varies. The cumulative cost over a ten year time horizon may increase by approximately 10% if second line is 50% more expensive than first line treatment. Loss to follow up is another challenge for ART programmes. Our simulations suggest that a significant proportion of new HIV infections may be caused by those who are lost to follow up. This will likely reduce the impact of the ongoing HIV treatment scale up. To increase efficiency, ART scale up programmes should incorporate strategies to reduce loss to follow up and minimise the risk of ART drug-resistance by monitoring and promoting ART adherence.

Keywords: HIV treatment; ART scale up; loss to follow up.

Potential impact of hormonal based contraceptives on HIV transmission dynamics among heterosexuals

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Abstract

HIV susceptibility linked to hormonal contraception has been studied before, but with mixed results. Recent findings have prompted the World Health Organisation, to encourage women who use hormonal birth control to concurrently use, condoms in order to prevent HIV infection in light of possible HIV risk of infection associated with hormone based contraceptives. A two sex HIV model classifying women into three risk groups consisting of individuals who use condoms, natural methods and hormone based contraceptives is formulated and analysed to assess the effects of various birth control methods on the transmission dynamics of the disease. Our model results showed that women who use hormonal contraceptives could be key drivers of the epidemic and that their increased infectivity together with increasing male partnership rates was pivotal in disease spread. Women who use hormone based contraceptives potentially act as a core-group from which men get infected and in turn transmit the disease to other population groups. We fitted the model to HIV prevalence data for Zimbabwe reported by UNAIDS and Zimbabwe Ministry of Health and Child welfare (MoHCW) and used the model fit to project HIV prevalence. Predictions using HIV data for Zimbabwe suggest that a hypothesised increase in susceptibility and infectivity of 2 fold would result in 25% increase in baseline HIV prevalence projection, thus suggesting possible increase in disease burden even in countries reporting plausible HIV prevalence declines. We conclude that the disproportionate spread of HIV within populations in sub-Saharan Africa may be explained by the methods of birth control used, thus compelling for the urgent need to reorient contraceptive methods used in settings with generalised epidemics.

Keywords: HIV; Birth Control; Hormonal Contraceptives; Mathematical Model.

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MODELLING THE SPREAD OF AIDS WITH TRANSMISSION FROM POOR CLINICAL SETTINGS IN MOZAMBIQUE

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Abstract

A HIV/AIDS model with heterosexual transmission and transmission from poor clinical settings is formulated and analyzed. The disease threshold parameters are computed, for which it is shown that the disease will die down if these initial threshold parameters are less than unity and will persist if they exceed unity. It is observed that even with a single sexual partner, the reproduction number is slightly greater than unity, implying that the additional transmission can only be from clinical settings. Stability (local and global) of both the disease-free and endemic equilibria are then investigated using various techniques of dynamical systems such as the Routh-Hurwitz Criteria and Lyapunov second method. Analysis on the bifurcation parameter is carried out to assess the impact of related HIV transmission from poor clinical settings. Estimates of the model parameters are obtained and used to numerically simulate the model represented graphically. Results show that HIV transmission from poor clinical settings contribute significantly to the spread of the disease.

An ecological agent-based simulation of regional innovation ecosystems

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Abstract

Regional governments under significant budgetary constraints face challenges developing and implementing policies to stimulate economic growth through the development and maintenance of innovation ecosystems. Innovation ecosystems comprise the set of public, entrepreneurial, and academic actors in a particular geographic region as well as the environmental conditions that mediate their activity. These systems are, by nature, complex, self-organizing, and adaptive. Although empirical studies and statistical models are valuable tools for evaluating the impacts of public policy on such systems, they focus on deterministic representations of system processes while avoiding social and environmental relationships. The ecological metaphor in particular brings forward concepts such as species diversity and robustness being key to ecological resilience, or the capacity of a system to respond to a disturbance by resisting damage and recovering quickly. In this work, we discuss our efforts to develop a computational model of a prototypical innovation ecosystem configuration that better captures system components missing from statistical models. We then proceed to detail our findings derived from systematic experimentation on an implementation of this model using Repast Symphony, an open-source, Java-based modeling and simulation platform. Lastly, we present a new predictive metric for measuring a region's resistance to economic disturbance; total regional technological resilience.

Estimating HIV treatment coverage in South African ART clinics using a mathematical model

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Abstract

In this study, we present a more inclusive method of estimating HIV treatment coverage in South Africa by incorporating the time-trend of the CD4 count distribution at ART initiation in the dynamic epidemiological model. We apply descriptive statistics to capture the time trend of the CD4 count distribution at ART initiation in the selected South African ART clinics. The statistical results have been used to calibrate the mathematical model to reproduce the number in need of treatment. Hence, the HIV treatment coverage.

Keywords:

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A Mathematical model for the treatment effects in the transmission dynamics of Plasmodium Malaria

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Abstract

A deterministic mathematical model for the dynamics of Plasmodium malaria in a population is formulated. The main objective of the study is to monitor the epidemiological impact of antimalarial drugs in reducing disease burden in a population where there is treatment failure. It was shown that the disease-free equilibrium point of the model is locally stable when the effective reproductive rate $R_{eff} < 1$. This disease-free equilibrium is also shown to be globally stable. The model also has a unique positive endemic equilibrium point for $R_{eff} > 1$. Sensitivity analysis of R_{eff} on the model parameters showed that treatment indeed reduce disease burden.

Stability of a stochastic model of an SIR epidemic with vaccination

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Abstract

We prove almost sure exponential stability for the disease-free equilibrium of a stochastic differential equations model of an SIR epidemic with vaccination. The model allows for vertical transmission. The stochastic perturbation is associated with the force of infection and is such as to keep the total population size remaining constant in time. The main result concerns especially the smaller values of the diffusion parameter, and describes the stability in terms of (at least partially) an analogue of the basic reproduction number \mathcal{R}_0 of the underlying deterministic model (without vaccination). Stability is proved beyond the range $\mathcal{R}_0 < 1$.

Keywords: Stochastic SIR model, basic reproduction number, vaccination, almost sure exponential stability.

A Mathematical Model for the Dynamics of the HIV Related Lymphomas (HRLs)

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Abstract

HIV is dramatically changing the demographics of diseases and the resource needs of the entire health care services of many countries in Africa. Malignancies such as the HIV related Lymphomas (HRLs) have an increased risk of developing in Persons Living with HIV (PLWH). In this project, a dynamic compartmental model detailing the co-infection dynamics of HRLs and HIV is formulated. In modelling the dynamics of HRLs, the population is divided into five subclasses i.e Susceptible $S(t)$ who are not yet infected but can be infected by HIV individuals through sexual contacts and develop lymphomas, HIV only infected individuals not yet displaying symptoms of AIDS $I_h(t)$, individuals developed lymphoma only $I_l(t)$, those infected with HIV and have since developed lymphoma $I_{hl}(t)$ and individuals with full blown AIDS $A(t)$. The model is broken into sub-models whose mathematical analysis is given. The co-infection model, that takes into account the sub-models is then investigated. The model is fitted to data obtained from the Tygerberg Lymphoma Study Group (TLSG) using the Matlab codes language. Projections will be made to depict the likely trends of HRL cases beyond 2012. This has health care implications on the planning and allocation of resources in many of the sub-Saharan countries in Africa. Sensitivity analysis will be performed to determine which parameters have the most profound effect on the outcome variables. For specific parameter values, the model is found to fit very well to the data that is available.

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A mathematical model for investigating Alan Savory's claims concerning management of savanna rangeland

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Abstract

Alan Savory proposes that desertification of savanna rangeland can be reversed, with little change in overall stocking rates, by concentrating the animals into large herds which allow the vegetation long recovery times in between short periods of very intense grazing [refernces needed!]. With a small amount of research and rather more debate and publicity, the proposal remains controversial [refs needed]. We propose a model for the interaction of vegetation, herbivores and soil resources that seems to capture Savory's hypothesis, and investigate whether there are parameter regions and stocking patterns that reproduce the claimed difference between constant stocking rate and transient high stocking rate.

Keywords: savanna, rangeland management, spatial effects in ecology, population dynamics.

Modelling drug epidemics, with saturated response functions, in the Western Cape Province of South Africa

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Abstract

Drug epidemics remain a global problem with immense health and social consequences. There has been a dramatic increase in treatment demand for drugs such as dagga, mandrax, cocaine, heroin and methamphetamine (MA) in the Western Cape Province (WCP) of South Africa in recent years. Drug abuse and the burden of drug use is also greater in the WCP when compared to other provinces in South Africa. Even though an increase in demand for substance abuse has been noted by many researchers, the actual number of people using drugs is not known. This is due to the fact that most data and information concerning drug abusers is found at rehabilitation centers or from household surveyors but the problem is that not every drug user goes for treatment at rehabilitation centers and also not every interviewed substance user comes clean when being interviewed because drug abuse is a criminal offense in the country. Therefore available tools which include mathematical models and collected data on drug users, will be used to estimate the prevalence of drug abuse.

Keywords: drug abuse; epidemics; saturation; response functions.

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Modelling effective HIV antiretroviral therapy in the liver

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Abstract

CD4+ and hepatocytes cells, both found in the liver, support all stages that lead to HIV production. Among people infected with HIV, liver disease has become the second most cause of morbidity and mortality. Considering HIV infection and replication in hepatocytes and CD4+ cells, a mathematical model is developed and analysed to investigate the ability of different combinational therapy to inhibit viral production in liver cells. Therapy efficacy in form of a dose-response function was incorporated. Analysis of the model shows that it is possible to have the effective reproductive number Re below unity provided the therapy efficacy is more than 90%. Within some range of parameter values, Re can also be reduced below unity at an efficacy of 50%. Simulation results show that combinational therapy of DDI, 3TC, ATV and NFV is the most effective while AZT, d4T, ATV and NFV is the least effective, in terms of inhibiting viral production. Findings showed that this model can be used to recognize which of the current treatment protocols perform best in controlling HIV replication in the liver.

Dealing with Household Correlations in using Survey Data to estimate Risk of Diarrhoea in Under-Five Children in Malawi

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Abstract

Statistical inference has played a vital role in estimating the proportion of children under great risk of diseases, such as diarrhoea. However, the data independence that is assumed alongside most classical methods is often empirically unattainable. Hence, statistical models that allow dependent observations have been developed. This study aimed at examining efficiency of such methods in extracting risk factors for under-five child diarrhoea in Malawi, relative to subject-specific models. To achieve this, random-effects, generalized estimating equations, survey models as well as subject-specific logistic and Poisson models were applied on the 2010 Malawi Demographic and Health Survey data. The analysis was carried out in Stata package, in which odds ratios were calculated for logistic and incidence rate ratios for Poisson models. The results showed that sizes of estimates and standard errors of survey models were generally lower compared to the rest of the models in all covariates except mothers pregnancy status and interaction of age and breastfeeding in which survey models had larger values. While the other three groups of models produced similar estimates throughout. However, the directions of the estimates from all the models were in agreement. The models showed significant association between diarrhoea and child's age, region of stay, and mothers literacy level, but disproved influence of child's sex, breastfeeding status, area of residence, mothers education level, mothers pregnancy status, and breastfeeding by a pregnant mother on the child's risk. The risk was high in children whose mothers were illiterate than those with literate mothers, children from central or southern region compared to northern region, and in children aged below 35 months than above. Further, the value of within-household variations was low in both logit and Poisson random-effects models ($\sigma_u^2 = 0.007$ and 0.002 , respectively), which implied that household observations were almost independent. Hence, survey model results were the most reliable and it has been recommended that researchers should consider applying survey models in cases where multilevel models reveal low intra-class correlations in the dataset and where the data at hand is from a large survey with known sampling structure.

Keywords: : diarrhoea, survey, intra-class correlation, random-effects, generalized estimating equations.

Modelling the Role of Diagnosis, Treatment and Health Education on Multidrug-Resistant Tuberculosis Dynamics

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Abstract

Tuberculosis, an airborne disease affecting almost a third of the world's population remains one of the major public health burdens globally, and the resurgence of multidrug-resistant tuberculosis in some parts of sub-Saharan Africa calls for concern. To gain insight into its qualitative dynamics at the population level, mathematical modeling which requires as inputs key demographic and epidemiological information can fill in gaps where field and lab data are not readily available. A deterministic model for the transmission dynamics of multi-drug resistant tuberculosis to assess the impact of diagnosis, treatment, and health education is formulated. The model assumes that exposed individuals develop active tuberculosis due to endogenous activation and exogenous re-infection. Treatment is offered to all infected individuals except those latently infected with multidrug resistant tuberculosis. Qualitative analysis using the theory of dynamical systems shows that, in addition to the disease-free equilibrium, there exists a unique dominant locally asymptotically stable equilibrium corresponding to each strain. Numerical simulations suggest that, at the current level of control strategies (with Malawi as a case study), the drug-sensitive tuberculosis can be completely eliminated from the population, thereby reducing multi-drug resistant tuberculosis.

Keywords: Tuberculosis model; diagnosis; treatment; health education; multi-drug resistant.

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Some insight given by Lie symmetry analysis on simple models in mathematical biology

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Abstract

Lie group analysis is arguably the most systematic vehicle for finding exact solutions of differential equations. Lie symmetry analysis has been applied successfully to many problems in different fields such as physics, ecology, finance etc. however the application of Lie group analysis to the models that frequently arise in the field of biology is relatively new. Using Lie symmetry analysis one can, for example reduce the problem of solving a complicated nonlinear system of differential equations to that of a single linear differential equation. In this talk we present the solutions via Lie symmetry analysis of particular differential equations model's drawn from epidemiology.

Keywords: Symmetry analysis, Mathematical biology, Lie group analysis, Mathematical modelling.

Bayesian Hierarchical Spatio-Temporal Modelling and Mapping of Tuberculosis in Kenya

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Abstract

This paper proposes to investigate and identify variation, clustering and spatio-temporal trend of tuberculosis' (TB) risk among Kenya counties from 2002-2009. In disease mapping, identification of spatial heterogeneity of disease risk or clustering investigation is often restricted to a single time period, but data in public health are often available for time window for several years, hence disease risk are variable in space and time, and supporting risk management should adopt spatio-temporal analysis approaches. However, implementation of spatio-temporal models as a quick response tool are quite difficult. Also, description temporal and spatial evolution of risk simultaneous in appropriate way has not yet been widely agreed upon. Therefore an in-depth effort is required to propose a model suitable for capturing and explaining TB phenomenon in Kenya. The data set obtained from Kenya's DHS, contains records of Kenya's population size, tuberculosis cases, and some suspected determinants of tuberculosis for each period from 2002-2009 and for each 67 districts. The district categorization was collapsed according to the current counties categorization. The Hierarchical Bayesian Spatio-Temporal approaches was used for modelling and mapping. Hierarchical Bayes approach provides smooth posterior relative risk by means of accounting for extra variability and borrowing information from neighbouring counties. Space and time interaction of risk therefore becomes an important factor in this paper since the measure of spatio-temporal models best performance largely depends on the specification of the interaction term ψ_{ij} . The Bernardinelli et al (1995) parametric model, the Waller et al (1997) independent model, and the Knorr-Held (2000) smooth temporal evolution model are considered for space-time modelling and mapping of tuberculosis in Kenya. Markov Chain Monte Carlo (MCMC) method via WinBUGS and R packages was used for simulations, and the Deviance Information Criterion (DIC), proposed by Spiegelhalter et al. [2002], was used for models criticism. Variation in TB risk was observed among Kenya counties and clustering among counties with high TB relative risk (RR). HIV was identified as a dominant determinant of TB. The risk of tuberculosis varies over the study period with increasing trend from 2009. Interaction of risk in space and time was found to be increasing in urban counties and decreasing in rural counties. We have recognised the existence of unknown space effect which is not persistent over the study period. Although the approaches are less than ideal, we hope that our formulations provide a useful stepping stone in the development of spatial and spatio-temporal methodology for the statistical analysis of risk from TB in Kenya.

FACTORS THAT PREDICT ATTENDANCE TO A MALARIA CANDIDATE VACCINE (RTS) STUDY AMONG CLIENTS IN LILONGWE, MALAWI

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Abstract

The malaria candidate vaccine (RTS) studies are in licensure stage in Malawi. Common to most clinical trials is the problem of missing data, where clients miss out some visits or drop off after a few visits. It was observed that some clients participating in the University of North Carolinas (UNC) RTS study missed out some clinic visits. This problem has the potential to prejudice the credibility of the final study findings and also reduce medical care given to clients who need it in course of the study. This investigation was therefore set to determine factors that predict attendance to the UNC's RTS study with the aim to suggest ways that may help to improve on clients continued attendance to such trials.

The study used primary data collected using questionnaires through a cross section study design. The study subjects were parents/guardians for children who were participating in the UNC's RTS study. The study enrolled 166 parents/guardians to respond to various items in the questionnaire related to attendance/non-attendance to the trial. Data was analysed using descriptive statistics and binary logistic regression to exploit factors that may predict attendance to the clinical trial.

The findings suggest that attendance among older children is becoming lower than among younger children. In addition, attendance among parents/guardians who understood the next appointment for the children was higher than among those who did not. These results point to two critical policy dimensions: (a) researchers or trial managers should strive to encourage parents/guardians with older children to continue attending clinic visits, if we are to achieve trial results that are representative across all ages, (b) trial managers should regularly keep on making sure that every parent/guardian understands next appointment for his or her child. **Keywords:** Attendance;

Malaria candidate vaccine (RTS); Binary logistic regression.

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A Statistical Analysis of Voluntary Counseling and Testing (VCT) Data to Determine the Risk Factors for HIV Infection in Namibia

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Abstract

In an effort to provide information that can guide prevention strategies, this paper determined the risk determinants of HIV infection in Namibia and estimated the potential for HIV prevention using attributable risk fractions. The study adopted a cross sectional research design with a sample of 14296 VCT clients from Oshana, Khomas and Kavango regions of Namibia for the period of 2009 to 2012. A multivariate logistic regression analysis was used to compute relative risks-measures of disease risk associated with various factors. For risk factors which are amenable to intervention, the Population Attributable Risk Percent (PAR %) was computed. The magnitudes of PAR% were then used as the basis for selecting risk factors that pose the greatest health threat to the population for prioritisation in combination HIV programming. From a targeting perspective, sex, condom use, marital status, region of residence, male circumcision status, age and level of education were found to be significant predictors of HIV infection. Alcohol use was not associated with HIV infection in these regions. Among these risk factors, not using condoms and not being circumcised for men are amenable to intervention and interventions that eliminate exposure to these risk factors can avert up to 22% and 19% of the disease burden respectively assuming all other conditions remain the same.

Keywords: statistical modelling, multivariate logistic regression, population attributable risk, HIV prevention.

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Impact of exogenous reinfection on TB infection in a genetically susceptible population

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Abstract

The development of TB from dormant state to active state is due to either endogenous reactivation or exogenous reinfection of a susceptible individual. The contribution of exogenous reinfection in TB proliferation in a genetically susceptible population has been scarcely studied. In this study we investigated the impact of exogenous reinfection on a genetically resistant and genetically sensitive sub populations. We qualitatively analysed the dynamics of TB by assuming that TB is transmitted in two ways namely homogeneous and heterogeneous mode of transmission. Analytically we computed the fundamental threshold used to measure disease persistence: basic reproduction number R_0 and found that the exogenous reinfection parameters fail to appear in basic reproduction number. Hence, basic reproduction number derived in presence of exogenous reinfection does not adequately predict the course of a TB epidemic. We obtained the exogenous reinfection threshold which indicated that exogenous reinfection complicates TB dynamics. Both analytical and simulation results disclosed that when exogenous reinfection is above exogenous reinfection threshold TB dynamics were governed by a backward bifurcation implying TB may continue to invade the population despite basic reproduction number being less than one. We computed critical value of basic reproduction numbers R_c and found that TB can only be eradicated if basic reproduction number is reduced below critical value R_c . Further, we incorporated TB therapy in heterogeneous model among individuals with clinically active TB and performed sensitivity and uncertainty analysis using Latin Hypercube Sampling. The sensitivity and uncertainty results showed that transmission rates, reactivation rates and proportion that is genetically resistant greatly influence variable outcome of our TB model.

A mathematical analysis of the co-infection of cervical cancer and HIV/AIDS in the presence of intervention

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Abstract

Cervical cancer is the leading cause of deaths in women in Zimbabwe and the world at large. The magnitude effect of the pandemic has been given less attention as compared to the prioritisation of diseases such as HIV/AIDS, TB and many other infectious diseases. We create two deterministic sex models for the co-infection of cervical cancer and HIV/AIDS within a community in the presence of intervention and rigorously analyze the models. We critically analyse the dynamics of the coinfection of cervical cancer and HIV/AIDS especially in a setup where resources are limited and evaluate the impact of intervention on such a system. The fact that the two diseases highly affect the immune system poses great challenges in any health system since a lot has to be taken into consideration in order to eradicate or reduce the prevalence of the disease. It is well known that cancer is one of the most expensive health problems within any developing country and hence the concept of prevention being better than cure plays a pivotal role in the study. We calculate the basic reproduction number R_0 , the effective reproduction number R_T , the disease-free, endemic and co-infection equilibria under specific conditions and show that in the presence of intervention $R_T < R_0$. We determine the stability criteria for the disease-free and endemic equilibria using the Routh Hurwitz criterion and the Lyapunov functional approach. We establish that both models are locally and globally stable at both equilibrium points. Numerical simulations show that co-infection exists and can be minimised if vaccination and treatment are effectively administered within a community.

Keywords: Cervical cancer, infectious, immune system, intervention, co-infection, Lyapunov, Routh Hurwitz criterion

4 DIF: Differential Equations

ON THE QUALITATIVE BEHAVIOUR OF SOLUTIONS FOR A CERTAIN CLASS OF THIRD ORDER NONLINEAR DELAY DIFFERENTIAL EQUATION

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Abstract

By using the frequency domain method, sufficient conditions which guarantee asymptotic stability of the null solution of a certain class of third order nonlinear delay differential equation are established. Furthermore, effective criteria for the existence of a bounded solution which is exponentially stable, periodic or almost periodic according as the forcing term is periodic or almost periodic are obtained. Our results generalize existing results in the relevant literature.

Keywords:third order nonlinear delay differential equations; bounded solutions; asymptotic stability; exponential stability; periodic solutions; almost periodic solutions; frequency domain.

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Modelling via Lie symmetry analysis: Some important recipes for solving differential equations

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Abstract

Many important mathematical models that arise in scientific investigations involve (systems of) differential equations, many of which are nonlinear and at best difficult to solve. Lie symmetry analysis represents perhaps the most systematic approach to solving and/or studying many such problems. In this talk some recipes of Lie symmetry analysis are presented and applied to familiar differential equation models that arise from various modelling contexts.

5 EDU: Mathematics Education

Mobile-Learning: Transforming Students' Attitude Towards Mathematics

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Abstract

Mathematics is one of the core subjects in Malawi secondary school curriculum and as such it is a prerequisite for admission into institutions that offer scientific studies. Negative attitude towards mathematics is a challenge to students' performance. Mitigating negative attitudes towards mathematics would encourage students to participate fully in the subject and consequently lead to improved learning of mathematics. Recognizing the implications of attitudes towards mathematics, this paper investigates the influence of mobile-learning in transforming students' attitudes towards mathematics. In developed countries, mobile phones have been used to create an environment in which mathematics learning and scientific practice have been perceived by students as activities that are worth using. The flexibility of Mobile-Learning has also made teaching and learning in those countries enterprising and enjoyable by both the students and the teachers. While mobile technology has been used in Malawi's health sector, its usage in academic sector is unknown. This study pioneers research in the use of mobile phones in teaching and learning of mathematics in Malawi schools. The study has employed a mixed methods approach and an experiment is being conducted in secondary schools in Chiradzulu district which is one of the lowest mathematics performing districts in Malawi. The study used purposive sampling in identifying its participants. The preliminary study involved 244 students while only 85 students are taking part in post-test study because it requires those with mobile phones to take part in mobile-learning. The initial findings show that 24.61% of the participants are comfortable with mathematics while the rest find mathematics more scaring and dreadful and they do not want to study it beyond secondary level. The study has revealed that most students have a negative attitude towards mathematics. With almost every student satisfied with the teachers' competence, the study therefore recommends that a technological intervention be introduced to rescue students deteriorating feeling about mathematics. **Keywords:** Attitude, Mathematics, Mobile Learning, SMS, Education.

6 EXA: Analytical solutions

New exact solutions and conservation laws of a coupled Kadomtsev-Petviashvili system

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Abstract

In this talk we obtain exact solutions of a new coupled Kadomtsev-Petviashvili system, which arises in the analysis of various problems in fluid mechanics, theoretical physics and many scientific applications. Lie symmetry method along with the multiple-exponential method is employed to find the travelling wave solutions of the underlying system. In addition, we derive the conservation laws of the coupled Kadomtsev-Petviashvili system using the multiplier method.

ANALYTICAL SOLUTIONS AND CONSERVATION LAWS OF THE GENERALIZED DOUBLE COMBINED SINH-COSH-GORDON EQUATION

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Abstract

A second order nonlinear wave equation, namely, the generalized double combined sinh-cosh-Gordon equation, which appears in a diverse range of physics such as solid state physics, non-linear optics and stability of fluid motion is studied. Lie symmetry analysis along with Exp-function method is used to obtain travelling wave solutions for this equation. Thereafter, conservation laws will be obtained by employing the direct method.

Keywords: Generalized double combined sinh-cosh-Gordon equation; Exact solution; Exp-function method; Conservation laws.

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Lagrangian and Hamiltonian approach in the mathematical analysis of the cardiac excitation propagation

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Abstract

The diffusion-reaction equation has been widely used as the governing equation for a host of biological phenomena, but its subsequent mathematical analysis often faces many limitations due to its non-conservational characteristics. In the past years, this restriction was applied similarly in analyzing the electrical signal propagation in the human brain and heart. In order to overcome this problem, a new theory has been proposed to show that the diffusion-reaction equation is just an equation for one scalar variable in Maxwell's equations of two electromagnetic field variables that is constructed in the biological bidomain. Then, Lagrangian and Hamiltonian approach can be easily pursued with the constructed Maxwell's equations to equip us with better mathematical tools to trace the trajectory of the electrical signal and to understand the excitation mechanism of the heart for the sake of improving surgical and clinical planning for cardiac patients.

SOLUTIONS OF THE SYMMETRIC REGULARISED LONG WAVE EQUATION

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Abstract

In this work we study the symmetric regularized long wave (SRLV) equation given by

$$u_{tt} - u_{xx} + \frac{1}{2}(u^2)_{xt} - u_{xxtt} = 0,$$

which arises in several physical applications, including ion sound waves in a plasma [1]. We use Lie symmetry methods along with the simplest equation method to obtain exact solutions of this equation.

Keywords: Lie symmetry methods, simplest equation method.

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Symmetry analysis and invariant solutions of the Fisher equation with time-dependent coefficients

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Abstract

We perform Lie symmetry analysis and derive some exact solutions of a time-variable coefficient Fisher equation which models reaction-diffusion-convection phenomena in biological, chemical and physical systems. These time-dependent coefficients (model parameters) are specified via group classification approach.

Keywords: Fisher equation; symmetry classification; exact solutions.

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7 FIN: Mathematical & Mathematics of Finance

Dividend maximisation under a ruin constraint in a surplus process compounded with a constant force of interest.

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Abstract

We consider an insurance company whose surplus dynamics are modelled by the perturbed classical risk process. The company is allowed to take advantage of a positive constant force of interest out there by investing into a risk free asset. We maximise expected discounted dividend payouts to shareholders under a predetermined ruin constraint using a barrier strategy. In this paper, the Volterra integral equations have been derived and solved using block by block methods. We have established the optimal barrier to use to pay dividends provided the ruin probability is no larger than the predetermined tolerance.

Keywords: Dividends; Volterra integral equation; HJB, Block-by-block method; Ruin Probability.

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Recursive utility and disappearing puzzles for continuous time models

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Abstract

Motivated by the problems of the conventional model in rationalizing market data, we derive the equilibrium interest rate and risk premiums using recursive utility in a continuous time model. Two ordinally equivalent versions are considered, as well as a heterogeneous model. The state prices depend on past values of the key economic variables in both versions, so instead of using dynamic programming we use the stochastic maximum principle. The resulting equilibriums are consistent with low values of the parameters of the utility functions when calibrated to market data. One version is consistent with preference for early resolution of uncertainty, the other for late for the US-data. We therefore consider heterogeneity with recursive utilities. Our resulting model rationalizes data well, and can explain both the Equity Premium Puzzle and the Risk-Free Rate Puzzle with good margins. We also consider limited market participation, as well as optimal asset allocation within this framework.

Optimal Portfolio Choice When Stock Price is a Semimartingale

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Abstract

We consider an optimal portfolio choice problem for an investor under different utility functions when the stock price process is a general semimartingale, facing imperfectly hedgeable stochastic income. The returns on the income and the stock are imperfectly correlated, therefore the market is incomplete. We describe how an investor adjusts the Merton portfolio of the stock and risk-less asset through an interpolating hedging demand, in reaction to the stochastic income.

Keywords: semimartingale; stochastic income; stochastic control; utility functions.

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Capacity building in Financial Mathematics: The case of Lesotho's Banking Sector as an Emerging Market

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Abstract

Financial mathematics as the application of mathematical concepts to financial situations can be used to model derivatives pricing and risk evaluation among other financial operations. All financial institutions are faced with a risk of some kind, such as operational risk, market risk, credit risk, liquidity risk, asset-backed risk, model risk and foreign investment risk. It was the present researcher's hypothesis that to manage risk and trade adequately, the banks need to develop their quantitative and mathematical skills. In particular, the study focused on how the Central Bank of Lesotho and the commercial banks that it supervises train their staff in financial mathematics. The conclusion is that there are more challenges (than successes) such as lack of applicability of financial mathematics concepts and the cost of training in capacity building of financial mathematics.

Keywords: Banks; risk management; emerging market.

Minimizing the Probability of Ultimate Ruin by Excess of Loss Reinsurance and Investments

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Abstract

We consider a risk management strategy where the insurance company chooses re-insurance under excess-of-loss reinsurance arrangement and investments into both risky and risk free assets.

We model the wealth dynamics of an insurance company by a risk process perturbed by diffusion. This process is then compounded by another return on investment process of Black-Scholes type. These two processes combined, form the risk process used in this work.

The HJB equation for this problem is then derived as well as its corresponding Volterra Integral Differential Equation of the second kind which is then transformed into a linear Volterra Integral Equation of second kind. We solve this integral equation numerically using the block-by-block method for different retention levels for chosen parameters.

The results show that Excess-of-loss reinsurance improves the survival of the insurer.

Keywords: Ruin Probability; Excess-of-loss reinsurance; Investments; Volterra integral equation; HJB, Block-by-block method.

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Experimental economics and bounded rationality

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Abstract

In this paper we establish a link between probabilistic cost efficiency and bounded rationality in the newsvendor model. This establishes a framework where bounded rationality can be examined rigorously by statistical methods. The paper offers a relatively deep theoretical analysis of underorders/overorders in the newsvendor model. The theory is supported by empirical findings from our analysis of empirical data from laboratory experiments. In particular, we observe that underorders are systematically larger than overorders, an issue that our theoretical model explains. From statistical tests we conclude that all variability in our data can be explained by probabilistic cost efficiency when critical fractiles are either small or large, but this does not hold in the middle range. The paper extends the results from Su (2008) in several new directions.

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A Mathematical Model of the Optimal Premium Policy of an Insurance Firm with Delay

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Abstract

In this work, we study the optimization problem confronted by an insurance firm whose management can control its cash-balance dynamics by adjusting the underlying premium rate. The firm's objective is to minimize the total deviation of its cash-balance process to some pre-set target levels by selecting an appropriate premium policy.

We study the problem in a general framework and make two interesting inclusions; Firstly, we introduce the aspect of time delay to the system; and we replace the standard expected additive utility function with a Stochastic differential utility (SDU).

We reformulate the problem and obtain a semi coupled Forward-Backward Stochastic Differential Equation and prove that this forward-backward system admits a unique solution. The Sufficient Maximum Principle for an optimal control of such a system is established and we conclude with a case study of three particular cases which fit into our general model. Some concluding remarks are also given.

Conditioned Stochastic Differential Equation and Modelling of Financial Information

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Abstract

In this paper, we generalize the notion of Brownian bridge market information model of Brody et al. [1, 2] to include other members of the family of Conditioned Stochastic Differential Equations (CSDE) associated with the conditioning of a marginal law. Specifically, the flow of information to the market is modeled as the the filtration generated by the solution $(\xi_{tT})_{0 \leq t < T}$, of a CSDE associated with a conditioning at the terminal point. Given the conditioning, (T, X_T, ν) and the initial condition $\xi_{0,T} = 0$, the law of $(\xi_{tT})_{0 \leq t < T}$ is the minimal probability associated with (T, X_T, ν) . We consider an asset with a payout of X_T at T. The information about X_T is then modelled by a CSDE with terminal value $\xi_{TT} = X_T$, and marginal law, ν at T. We derive the price process of the asset along with prices of options for three different specifications of ν . We also introduce the idea of modeling market information by CSDEs associated with the quadratic variation of Brownian motion and CSDEs associated with the first hitting time of a level, which are appropriate for the modeling of information regarding cash flow of perpetual American options and Barrier options respectively.

Keywords: Brownian Bridge; stochastic differential equations; information process; markovian bridge;

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On the Price of Risk Under Regime Switching Exponential Lévy Model: A Case of CGMY Process.

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Abstract

We study the pricing of an option when the price dynamics of the underlying asset are governed by a regime switching exponential Lévy process. We suppose that the market parameters, for instance, the market interest rate, appreciation rate and volatility of the underlying risky asset are time dependent and are modulated by a continuous time, finite state hidden Markov chain. We first study the problem in a general framework, that is, a general regime switching exponential Lévy model. In general, the market described by regime switching exponential Lévy models is incomplete and hence the equivalent martingale measure (EMM) is not unique. We adopt the regime switching Esscher transform in order to determine an equivalent martingale pricing measure. This regime switching Esscher transform is taken conditional on the information about the Markov chain and as a result, we obtain two families of EMMs. We study two families of EMM for a general regime-switching Lévy model and this allows us to assess the price of risk. We use the minimisation of maximum entropy between an EMM and the real world measure over the different states in order to select appropriate Esscher parameters. We reformulate the problem to particular cases and study the regime switching Black-Scholes, regime switching jump diffusion and finally the regime switching CGMY model. Numerical experiments are conducted and their results reveal that the impact of pricing the regime switching risk on the price of an option is significant.

Keywords: Option pricing; regime-switching risk; regime switching Esscher transform; martingale restriction; min-max entropy problem.

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Computation of Greeks for Heston model using Malliavin calculus

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Abstract

Investors make predictions on how option prices vary over a certain period of time basing on the past market events. In general, the vast number of different market events makes it a difficult task. It is, therefore, important to understand which factors contribute to the movement of prices and with what effect. This sensitivity analysis is carried over parameters appearing in models for price dynamics and the so-called Greeks represent a form of measure for price sensitivity to some factors. In this paper we investigate the application of Malliavin calculus to the computation of Greeks for Heston model. We consider both the case without jumps and the case with jumps.

Keywords: Malliavin calculus; Greeks; Malliavin derivative; Model parameters; Bismut-Elworthy-

Li formula

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Optimal financing and dividend control of the insurance company with proportional reinsurance policy in a Lévy market.

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Abstract

We extend the results in He and Liang (2008) to the Lévy market, where the optimal control problem of the insurance company with proportional reinsurance policy was considered under diffusion models. A company's management controls the reinsurance rate, dividends payout as well as the equity issuance processes to maximize the expected present value of the dividends minus equity issuance until the time of bankruptcy. The resultant mixed singular-regular control problem is solved by decomposing it into two categories of suboptimal models, one is classical model without equity issuance, the other never goes bankrupt by equity issuance.

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LIBOR Market Model Versus the Levy LIBOR Market Model

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Abstract

In this thesis, we outline the numerical implementation of Lévy-LIBOR framework. We first look at the construction of LIBOR Market Model (LMM), address the major problems associated with specifically the drift component of LMM. Due to the complexity of the drift for LMM, Monte Carlo method seems to be the ideal tool to use. However, Monte Carlo method is time consuming and therefore an expensive tool to use. To improve on the process we look beyond the dynamics of the lognormal distribution, where Brownian motion (the only Lévy process with continuous paths) is the driving process and apply other Lévy processes with jumps as the driving process in the dynamics of LIBOR. The resulting process is called Lévy LIBOR Model constructed in the framework of [1]. The Lévy LIBOR model is a very flexible and a general process to use except with a complicated drift part in the terminal measure. The complicated drift term is as a result of random terms in the drift part as a result of change of measure. We use Picard approximation in the resulting drift component to make the processes tractable in the framework of [3]. With this approximation we expect the parameters of LIBOR Model to be more consistent with the market dynamics, with the introduction of the jump processes.

Keywords: LIBOR; Lévy Processes; Picard approximation;

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On pricing kernels, information and risk

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Abstract

We discuss the finding that cross-sectional characteristic based models have yielded portfolios with higher excess monthly returns but lower risk than their Fama-and-French arbitrage pricing theory counterparts in an analysis of equity returns of stocks listed on the JSE. Under the assumption of general no- arbitrage conditions, we argue that evidence in favour of characteristic based pricing implies that information is more likely assimilated by means of nonlinear pricing kernels for the markets considered.

Keywords: Arbitrage pricing theory, characteristic based models, size effect, value effect, linear pricing kernel, nonlinear pricing kernel

ANALYSIS OF OPTIMAL RETURN ON INVESTMENT IN SOME NIGER DELTA OIL FIELD PROJECTS: USING SUBORDINATED LÉVY PRICE PROCESS

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Abstract

The huge financial involvement in oil projects demands that the project manager takes no chances with optimality of his decision. Oil field development projects face market risks largely because the parameter of key importance, the oil price, fluctuates rapidly over time. The decision to invest or not in an oil field project is therefore very challenging, because information concerning the field is often scarce. Neither the future production, nor sales prices are known with certainty. Constraints on production level also exist in addition to OPEC quota, which is used in this work to proxy for other production limitations.

Price and output level determine return on investment in the project. Price data is calibrated using MLE, to obtain parameters of the processes under study. The price process that best describes the fluctuations in oil price is expected to yield better analysis with respect to expected returns (viability of the project).

Stochastic volatility models, such as Lévy-driven processes categorize price as a random variable and capture small and frequent jumps. Thus Subordinated Lévy processes such as Variance Gamma and New Inverse Gaussian are considered in the analysis, with emergence of interesting results. The process is identified and it provides a good basis for price forecasting, and optimality of investment decisions.

Keywords: Lévy Process, Brownian motion, Oil field, Expected Returns, Oil Price.

Pricing of European options when the Stock Price Process is Being Driven by Geometric Brownian Motion

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Abstract

In calculating the price of an option the volatility parameter plays a vital role, we first estimate this parameter using an example of a basic estimator and simulated values of geometric Brownian motion (gBm), hence explored some of the properties that improves the accuracy of an estimator. The theory was then extended to estimate the volatility from real data by using the Roger-Satchell(1994) Estimator. Hence the estimated volatility was used in the model developed by Aase. K, Oksendal. B. and Uboe. J(1998) in the Donsker Delta Function approach in calculating the value on European option and was compared with that of the Black-Scholes(1973) formula. A unique finding is an observation that the value of an option obtained from using the Donsker Delta Function approach is more of the European Put Option than European Call Option which were using the Black-Scholes formula, then this roughly leads to the conclusion that the Donsker Delta Function approach computes a European Put Option.

Keywords:Option Pricing; Donsker Delta function; Black-Scholes; Geometric Brownian Motion.

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Pricing measures and contingent claims classification in incomplete markets

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Abstract

In this talk, we revisit pricing contingent claims in incomplete markets. While a lot has been done on pricing in incomplete markets, there is still a gap on the categorization of the payoffs. Some contingent claims are attainable while others will not be attainable. We address the question of which contingent claims belong to each group. We also propose a generalization of the equivalent martingale measures used for pricing, a generalization which includes those studied so far. We also provide some examples of how to price in each class and introduce important definitions and to also address the question of lack of uniqueness of some of the pricing measures.

Optimal execution with multiplicative price impact

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Abstract

We consider the so-called “optimal execution problem” in algorithmic trading, which is the problem faced by an investor who has a large number of stock shares to sell over a given time horizon and whose actions have impact on the stock price. In particular, we develop and study a price model that presents the stochastic dynamics of a geometric Brownian motion and incorporates a log-linear effect of the investor’s transactions. We then formulate the optimal execution problem as a two-dimensional degenerate singular stochastic control problem. Using both analytic and probabilistic techniques, we establish simple conditions for the market to allow for no price manipulation and we develop a detailed characterisation of the value function and the optimal strategy. In particular, we derive an explicit solution to the problem if the time horizon is infinite. Interesting features of the problem’s solution include the facts that (a) the value function may be discontinuous as a function of the time horizon and (b) an optimal strategy may not exist even when the value function is finite.

Keywords: Optimal execution problem, multiplicative price impact, singular stochastic control.

MAXIMAL EXPONENTIAL INEQUALITIES FOR BESSEL PROCESSES

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Abstract

In this talk we extend some well-known power-type maximal inequalities for Bessel processes to the exponential case. Our method of proof is essentially based on the optimal stopping theory, maximality principle and a comparison result for a system of first-order nonlinear differential equations.

Keywords: Bessel processes; maximality principle; optimal stopping problem.

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Optimal portfolio strategy with discounted stochastic cash inflows where stock price is a semimartingale

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Abstract

We look at portfolio strategy with discounted stochastic cash inflows (SCI). The market is characterized by inflation bond, stock and bank account where the inflation bond and the stock are stochastic and the bank account is deterministic. The stock price is a semimartingale jump diffusion model. The price density is given by the Dolean's Dade exponential.

Keywords: Portfolio; semimartingale; jump diffusion; Dolean's Dade.

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8 FLU: Fluid Mechanics

THE WELL-POSEDNESS FOR THE EQUATORIAL HYDRODYNAMIC MODEL. A CASE STUDY OF LAKE VICTORIA.

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Abstract

A shallow water three dimensional hydrodynamic model has been applied to an Equatorial water body of Lake Victoria. With vertical coriolis as a dominant centripetal mass force, we test the well-posedness in order to apply a mathematical numerical finite volume method to capture both the physical and numerical shocks in the shallow water. Some hydrodynamic approximations have been considered to simplify solutions. An energy method is applied for the well-posedness of this non-linear hyperbolic system of navier-stokes equations

Double Diffusive Heat and Mass Transfer Over a Vertical Plate in the Presence of Wall Suction and Chemical Reaction

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Abstract

A steady incompressible boundary layer flow over a permeable vertical plate in the presence of a chemical reaction and wall suction is investigated. The governing fluid flow equations are transformed into a set of coupled ordinary differential equations with the help of similarity transformations and solved using asymptotic approximations in the presence of large buoyancy to obtain closed form solutions of the skin friction, Nusselt and Sherwood numbers. The effects of varying the buoyancy parameter on the velocity, concentration, temperature, skin friction and the rates of heat and mass transfer are determined and presented graphically. Results indicate that an increase in buoyancy is accompanied by an increase in fluid velocity and a decrease in the fluid temperature and fluid concentration. Results also show that an increase in buoyancy is accompanied by an increase in skin friction, while the rates of heat and mass transfer fall rapidly from very large values close to the wall down to a minimum value and then start to increase as the buoyancy parameter becomes larger. It is also noticed that the increase of the rate of heat transfer is more pronounced than the rate of mass transfer as the buoyancy parameter is increased.

Keywords: Double Diffusive Convection, Mixed Convection, Boundary Layer, Buoyancy, Wall Suction, Skin Friction.

Asymptotic behavior of Stokes flow in a thin domain with a moving rough boundary

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Abstract

The Reynolds equation lays the foundations of lubrication theory. It describes the flow of a viscous fluid in a thin 3D-domain bounded by two moving surfaces and is widely used by engineers to compute pressure and stresses in various fluid film bearings. As shown by numerous theoretical studies, the Reynolds equation can be obtained from the NavierStokes equation by asymptotic analysis when the film thickness approaches zero. In extremely thin films the effects of surface roughness upon the flow may be significant, hence the need for replacing the traditional Reynolds equation with a better model. To this end homogenization theory offers a cost-effective alternative. In the present study the fluid domain is described by two small parameters ϵ (related to the film thickness) and μ (the wavelength of the surface roughness). The flow is assumed to be governed by the evolution Stokes equation. Using the formal method based on two-scale asymptotic expansions we study the asymptotic behavior of velocity field and pressure as both ϵ and μ tends to zero. Depending on the ratio ϵ/μ three equations of Reynolds type are obtained in the limit. These equations have coefficients which are computed from solutions of local problems, thereby taking into account the surface roughness in an average sense.

MODELLING AND ANALYSIS OF SHEAR BANDED FLOWS OF COMPLEX FLUIDS

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Abstract

We present the Johnson-Segalman constitutive model to capture certain fluid flow phenomena that has been experimentally observed in the flow of complex polymeric fluids. In particular, experimentally observed phenomena such as shear banding, spurt and slip are explored and/or explained in terms of the non-monotonic shear-stress versus shear-rate relationships. We also explore the effects of the inclusion of stress diffusion into the stress constitutive model in terms of predicting shear-stress (or shear-rate) paths. We employ semi-implicit finite difference methods for all the solution processes.

Keywords: Johnson-Segalman Model; Diffusive Johnson-Segalman Model; Shear banded flow; Finite difference methods; Complex fluid flow.

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On a spectral perturbation method for unsteady boundary layer flow of a nanofluid past a stretching sheet

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Abstract

A spectral perturbation method for solving unsteady boundary layer flows is presented. The study seeks to improve solutions of nonlinear partial differential equations arising in fluid mechanics using perturbation methods. In the method, governing sequences of nonlinear partial differential equations generated by the perturbation series approximation are solved using the Chebyshev spectral method. To demonstrate computational efficiency, reliability and accuracy of the spectral perturbation method we solve the nonlinear partial differential equations governing the unsteady boundary layer flow of a nanofluid past a stretching sheet. The findings of the study were validated using the Keller-box numerical scheme.

Flow and reactive transport processes in porous media

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Abstract

Flow and reactive transport of chemical species is a very common phenomenon that occurs in natural as well as artificial systems. In this study, acid mine drainage in the South African context is considered. Due to the hazards associated with acid mine drainage, prevention or treatment of mine effluent before it is discharged into receiving waters and other environments is a necessity. A time-dependent mathematical model is developed for a passive treatment method, based on multi-scale modelling which couples physico-chemical processes involving diffusion, convection, reactions and filtration of chemical species. The time-dependent model is simulated on a two-dimensional domain using finite-volume discretisation to obtain concentration profiles of the chemical species.

9 GRA: Graph Theory

On the definition of Regular Clique Assemblies

Kelly Bragan* and Peter Johnson
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Abstract

A regular clique assembly is a regular graph G with positive degree satisfying: (1) every maximal clique in G is maximum, and (2) each edge in G belongs to exactly one maximum clique. We explore the conditions in this definition further; in particular, it is demonstrated that it is indeed necessary to require that every maximal clique be maximum.

Keywords: Regular Clique Assembly, Edge-Regular, Clique, Clique Graph, Friendship Graph

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Constructively Coloring the Line

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Abstract

Kemnitz and Marangio showed (2007) that for any set of k distances, the real line can be colored with $k + 1$ colors so that all k of the distances are forbidden, meaning that if two points have the same color, then the distance between them is none of those k distances. Their proof is non-constructive, as it appeals to a famous theorem of de Bruijn and Erdos that is a close relative of the Axiom of Choice. Here we give explicit instructions for obtaining distance-forbidding colorings in which each color set is a union of intervals, and we raise questions, which we believe to be open and non-trivial, about periodic colorings

Security in Graphs

Cadavious Jones* and Peter Johnson
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Abstract

The notion of security in graphs was first introduced and developed by Brigham, Dutton, and Hedetniemi in 2007. Given a graph $G = (V, E)$, a set $S \subseteq V$ is said to be *secure* if and only if every *attack* on S by vertices outside S can be successfully *defended*. We define *attacks* and *defenses* and note the short proof given by Isaak, Johnson, and Petrie in 2012 of Brigham, Dutton, and Hedetniemi's Characterization of secure sets. Our contribution is to use the principal ingredient of that proof, Halls Theorem (1935), to give efficient algorithms for finding a successful *defense* of an *attack*, when one exists.

A CHARACTERISATION OF ECCENTRIC SEQUENCES OF MAXIMAL OUTERPLANAR GRAPHS

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Abstract

Let G be a connected graph. The eccentricity of a vertex v is defined as the distance in G between v and a vertex furthest from v . The nondecreasing sequence of the eccentricities of the vertices of G is the eccentric sequence of G . In this talk, we characterize eccentric sequences of maximal outerplanar graphs.

Keywords: Planar graphs; Distance; Eccentricity.

Codes, graphs and designs from alternating groups A_n for n odd

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Abstract

Group actions on sets have played a key role in elucidating symmetry. They also have rendered themselves amenable to construction of various objects with a measure of symmetry guaranteed. In this paper we employ group actions of the alternating groups on $2k + 1$ sets to produce codes, graphs and designs. We explore codes, graphs and designs from alternating groups A_n for n odd. In particular, we look at the primitive action of the alternating group A_{2k+1} on $\Omega^{\{k\}}$, the k -subsets of $\Omega = \{1, 2, \dots, 2k + 1\}$. We investigate the binary codes obtained from the adjacency matrix of each of the k graphs with vertex set $\Omega^{\{k\}}$. In each case we find the parameters of the designs, codes and their duals. Further, the question of the automorphism groups of the combinatorial entities constructed will be investigated. We also show that a class of graphs we obtain from this enterprise are strongly regular, a rare phenomenon on graphs.

Keywords: Group actions; alternating groups; binary codes; designs; adjacency matrix; strongly regular graphs.

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The Hall ratio, the fractional chromatic number, and the Mycielski graphs

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Abstract

The Hall ratio of a finite simple graph G is the maximum, over all induced subgraphs H of G , of the ratio $|V(H)|/(\text{vertex independence number of } H)$. The Hall ratio is a lower bound of a large class of generalized chromatic numbers, including the fractional chromatic number. The fractional chromatic number equals the Hall ratio for a great many graphscycles, bipartite graphs, and complete multipartite graphs, for instance.

It is known that the difference of the fractional chromatic number and the Hall ratio can be arbitrarily large. What about their ratio? Therein lies an open question. We havent settled it (as of the submission of this abstract), but we have hopes for the Mycielski graphs.

Edge-Grundy Numbers of Complete Multipartite Graphs I

Ryan Matzke*¹ and Matthew DeVilbis ²
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Abstract

A Grundy edge-coloring of a graph is a proper (adjacent edges get different colors) assignment of positive integers to the edges of the graph such that if an edge is colored (assigned) $c > 1$, then the edge is adjacent to edges of all the colors $1, \dots, c - 1$. The edge-Grundy number of a graph is the largest positive integer appearing on an edge of the graph, among all the colors appearing on all the Grundy edge-colorings of the graph. The practical significance of this number is its function as an index of a worst possible outcome during online or greedy proper colorings of the graph; such colorings are useful in scheduling. In this work we determine the edge-Grundy numbers of all the regular complete multi-partite graphs, and give bounds on edge-Grundy numbers of some non-regular complete multi-partite graphs. This is the first presentation of a two-part series that will focus on introductory material and basic results.

Edge-Grundy Numbers of Complete Multipartite Graphs II

Matthew DeVilbis*¹ and Ryan Matzke²
University of Dayton¹, Gettysburg College²

Abstract

A Grundy edge-coloring of a graph is a proper (adjacent edges get different colors) assignment of positive integers to the edges of the graph such that if an edge is colored (assigned) $c > 1$, then the edge is adjacent to edges of all the colors $1, \dots, c - 1$. The edge-Grundy number of a graph is the largest positive integer appearing on an edge of the graph, among all the colors appearing on all the Grundy edge-colorings of the graph. The practical significance of this number is its function as an index of a worst possible outcome during online or greedy proper colorings of the graph; such colorings are useful in scheduling. In this work we determine the edge-Grundy numbers of all the regular complete multi-partite graphs, and give bounds on edge-Grundy numbers of some non-regular complete multi-partite graphs. This is the first presentation of a two-part series that will focus on introductory material and basic results.

10 NUM: Numerical Algorithms & Numerical Methods

Solving the Vibration of the current-carrying wire in a magnetic field using Variational Iteration Method

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Abstract

In this paper, Variational Iteration Method (VIM) is proposed to solve the dynamic oscillation of a current-carrying wire in a magnetic field generated by a fixed current-carrying conductor parallel to the wire. Two linear springs are considered to restrict the wire to a rigid wall. For a special case, the periodic solution of the problem is obtained by VIM and compared with numerical solutions for different parameters. The results show the high accuracy of this method and methods can be easily extended to solve other non-linear vibration equations and so can be found applicable in engineering.

Keywords: Current-carrying wire; Non-linear equation; linear elastic springs; Variational Iteration Method.

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The mixed finite element multigrid preconditioned conjugate gradient method for Stokes equations

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Abstract

This study considers the solution of the classical Stokes problem that models the steady state incompressible fluid flows by a fast solver. We study the coupled system of discrete equations emanating from the mixed finite element discretisation of the Stokes problem. The main thrust being the construction of efficient and robust iterative solution schemes. In particular, application of the preconditioned conjugate gradient method in a non-standard inner product for efficient solution of these problems. We investigate the scaled preconditioned conjugate gradient method in a non standard inner product with different preconditioner approximation combinations and present comparative results on the performance of these iterative schemes in terms of computational time and iterative counts. The preconditioned conjugate gradient without scaling proved to be worthless as it diverges. The results show that the the multigrid method is effective in accelerating the performance of the preconditioned conjugate gradient in a non standard inner product. The multigrid preconditioner combines very well with Chebyshev approximation of the Schur compliment. We also present the main theoretical convergence results. We study the problem in a two dimensional setting using the Hood-Taylor $Q_2 - Q_1$ pair of finite elements. The incompressible flow iterative solution software(IFISS) matlab toolbox is used to assemble the matrices. We present the numerical results to illustrate the efficiency and robustness of the preconditioned conjugate gradient scheme with the multigrid preconditioner.

Keywords: classical Stokes problem, mixed finite element method, multigrid, preconditioner ,non-standard inner product, non standard preconditioned conjugate gradient method.

Terminating Simplified Newton Iterations

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Abstract

The paper deals with the numerical solution of IVP's for systems of stiff ODE's with particular emphasis on implicit linear multistep methods (LMM), particularly the backward differentiation formulae (BDF). In this paper we investigate the current strategies that are used to terminate the Newton iterations in the Matlab Code `ode15s`. We analyze the algorithms for terminating the Newton iterations as implemented in the code `ode15s`. We then modify the existing termination strategy. Our numerical experiments reveal an improvement in terms of computational costs

Comparison of the EM algorithm and the Quasi-Newton method: an application to mixtures of developmental trajectories

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Abstract

To identify homogeneous clusters of individuals with similar developmental trajectories, a group based approach proposed maximizing the likelihood function using the Quasi-Newton method. The Quasi-Newton method has some convergence problems. To address these convergence problems the use of the EM algorithm is proposed. A comparison is made between the Quasi-Newton method and the EM algorithm. The results suggest that the EM algorithm has better convergence properties as it is less sensitive to starting values and less likely to converge to false maxima.

Keywords : developmental trajectory; mixture model; Quasi-Newton; EM Algorithm

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Functionally fitted Runge-Kutta-Nyström methods

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Abstract

We build on our previous results that showed that functionally fitted Runge-Kutta (FRK) methods can be studied using a convenient collocation framework. We extend this framework to functionally fitted Runge-Kutta-Nyström (FRKN) methods, stating that FRKN methods can integrate a second order initial-value problem exactly if its solution is a combination of certain functions, and that superconvergence can be obtained when the collocation points satisfy some orthogonality conditions. An analysis of their stability is also conducted.

Large On One-Dimensional Arbitrary High-Order WENO Schemes for Systems of Hyperbolic Conservation Laws

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Abstract

In this paper, a one-dimensional case for an arbitrary high-order non-oscillatory finite volume scheme is presented. This is an adaptation of the schemes presented in [1]. An entire polynomial inside each control volume is reconstructed in terms of hierarchical orthogonal polynomial bases over a reference element. A new numerical inter-cell flux function at the element interfaces is proposed. To validate the approach several one-dimensional test problems with discontinuous solutions are computed. The results show an improvement on the non-oscillatory shock-capturing properties based on the new approach.

Keywords: WENO reconstruction, Finite Volume Schemes, Conservation Laws

References

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ANALYSIS OF TWO-FACTOR EXPERIMENTS WITH MISSING VALUE AT RANDOM

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Abstract

In this talk we consider two-dimensional exterior problem of scattering of a time-harmonic acoustic incident plane wave by a smooth sound soft object Ω , with Lipschitz boundary Γ . Such a problem can be formulated by the Helmholtz equation:

$$\Delta u + k^2 u = 0 \quad \text{in } D := \mathbf{R}^2 \setminus \bar{\Omega} \quad (1)$$

with sound soft boundary condition and Sommerfeld radiation condition. Here $k > 0$ is the wavenumber, which is proportional to frequency. As k increases, corresponding to incident field oscillating more rapidly, so the complexity of the solution of (1) increases. As a result the computational cost of standard schemes will grow in direct proportion to k , leading to large computing times for large k .

In order to achieve an accurate solution of (1) for large k with reasonable computational cost, oscillatory nature of the solution must be taken into account when designing a computational method. Recent research has focused on enriching the approximation space with highly oscillating functions, such as plane waves or Hankel functions, in order to accurately represent the scattered field. Here we propose a collocation boundary element method to solve (1). We demonstrate via numerical experiments that the computational costs are substantially reduced as compared to standard schemes.

**A new interpolation approach for solving Partial differential equations that model
non-similar boundary layer flow**

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Abstract

In this work we present a new approach for solving nonlinear partial differential equations (PDEs) that model the problem of forced convection flow adjacent to a wedge. The proposed method is based on a coupling of interpolation based collocation method with quasi-linearization. Numerical simulations are conducted to generate results for the important flow properties such as velocity profiles and local skin friction. The accuracy of the present results is validated against existing results from literature and against results generated using the spectral quasi-linearization method. The preliminary results from the proposed study indicate that the present method is more accurate and computationally efficient than more traditional methods used for solving PDEs that describe non-similar boundary layer flow

The He's Variational Iteration Method and Its modification

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Abstract

The principles of the variational iteration method and its applicability for various kinds of differential equations are given in [1, 2, 3]. In this paper, we apply VIM for following nonlinear differential equation:

$$\mathcal{L}u(t) + \mathcal{N}u(t) = g(t), \quad (2)$$

where \mathcal{L} , \mathcal{N} are linear and nonlinear operators, respectively, and $g(t)$ is the source inhomogeneous term. According to the variational iteration method, we can construct the correction functional for Eq. 2 as:

$$u_{n+1} = u_n + \int_0^t \lambda (\mathcal{L}u_n + \mathcal{N}\tilde{u}_n - g(s)) ds, \quad (3)$$

where λ is the general Lagrange multiplier, which can be identified optimally via variational theory.

In this paper we discuss about VIM and its modification for solving nonlinear differential equations.

Keywords: Variational Iteration Method; Nonlinear differential equations; Lagrange multiplier.

References

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COMPUTATIONAL SOLUTION OF A 2-D STEADY NON-NEWTONIAN NATURAL CONVECTION FLOW

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Abstract

We focus on the steady two dimensional natural convection flow of non-Newtonian fluids. The governing mathematical equations for the steady flow are posed as a set of boundary value problems. This set of partial differential equations is however highly nonlinear and hence the relevant numerical techniques for this class of problems becomes quite cumbersome. We thus restate the set of governing equations under the time dependent framework leading to a set of initial-boundary value problems which can be easily solved via (semi-implicit) finite difference techniques. The transient finite difference solutions are used to recover the required steady solutions in the long time. The underlying problems find a variety of engineering and industrial applications.

Keywords: Natural convection; non-Newtonian fluid; finite difference method; Steady flow solution.

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A Glimpse at the Field of Hypersurface Singularities.

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Abstract

Singularity theory is a rapidly developing field in modern mathematics at the crossroads of a large variety of fields including algebraic geometry, commutative algebra and computational algebra. It has many applications in the natural and technical sciences, for example the theory of dynamical systems, geometric and physical optics, cosmology, chemistry, economics and many more. Those who love to have mathematics as their all day companion will recognise the singularity visible in their first cup of coffee at breakfast; the singularities making the waves glimmer at sea; the singularities responsible for making the prices of clothes and food you buy jump; and tell themselves that engineers, for sure, need to study more singularity theory when machines do things they do not predict!

In this talk we will have a glimpse at the field of hypersurface singularities. We will in particular discuss the problem of classifying these singularities and the big contribution by the famous Russian mathematician V.I. Arnold in this regard. Finally we will discuss how the use of computational algebra makes this field much more explorable. As an example in this regard we will present results by Andreas Steenpass and myself in this regard.

Keywords: Hypersurface Singularities; Algorithmic Classification; Algebraic Geometry.

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11 OPT: Optimization

On finding optimal routes for a distributor with time dependent travel times: A Vehicle Routing Problem.

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Abstract

This paper extends our previous work of finding relatively simple but optimal solutions to problems of transportation encountered by retail businesses. In the present paper, we consider both distance and travel times criteria. In our previous paper, only distance minimization criterion was considered in which the prim's algorithm was applied. However, in a practical setting, besides the goal of the producers and distributors to deliver goods to different customers in an efficient way, it is also important to consider the time variable which is sometimes uncertain. Motivated by this idea, in addition to distance minimization, we study the vehicle routing problem with soft time windows and time dependent travel times. For the latter, we propose to use a modified Dijkstra's algorithm to solve it. We perform computational experiments using a local bread distributor. We then compare results from the Prim's and modified Dijkstra's algorithms to obtain optimum routes in a time dependent setting.

Keywords:

Vehicle routing problem with time soft windows, Prim's algorithm, Dijkstra's algorithm, time dependent travel times.

References

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On Stable Matchings

Adam Blumenthal* and Peter Johnson
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Abstract

The Gale-Shapley Theorem, sometimes called the Stable Marriage Theorem, asserts that there is a stable matching in every finite locally ordered bipartite graph. We show that this statement characterizes bipartite graphs, among finite simple graphs.

12 SOL: Solid Mechanics

Modelling the Malawi heavy loader bicycle

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Abstract

Bicycles are used extensively in Malawi for transportation of goods and people. 41% of rural households in Malawi own a bicycle, according to the report of the Integrated Household Survey for Malawi for 2010. Bicycles sold in Malawi originate from China and India. These bicycles are then modified to carry heavy goods.

We follow the analysis in [1] and [2] to model the Malawi heavy loader bicycle for small lean and steer angles. We compute the dynamics and perform an eigenvalue analysis and present our results.

Keywords: Bicycle stability, lean angle, steer angle, quadratic eigenvalue problem

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Plane elasticity problem involving regions of a circular domain

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Abstract

The Elasticity field equations are very complex and that is why analytical-closed form solutions to fully three-dimensional problems are very difficult to accomplish. This is why most solutions are developed for reduced problem that typically include axisymmetry or two-dimensionality to simplify particular aspects of the formulation and solution. In this paper, we show some solutions of particular plane elastic problems involving the regions of circular domain. The process starts by developing a general solution to a circular region with arbitrary edge loading. The specified region will have an arbitrary boundary loading specified by stress conditions. The solutions of the fundamental stress combinations and displacements are given in polar coordinates. We illustrate the results with two examples.

13 STA: Mathematical Statistics

THE M/M/1 QUEUE IN A VIOLENT SPACE

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Abstract

We study the M/M/1 queueing system located in a zone where conflict occurs with a positive probability. It is assumed that a conflict process may emerge at any random time t when a stationary set of customers are present in the system. For $\delta > 0$ called the conflict probability when the system is busy, we obtained expressions for the randomized queue length distribution and compute relevant performance measures.

Keywords: The M/M/1 queue; conflict probability; queue length distribution.

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ANALYSIS OF TWO-FACTOR EXPERIMENTS WITH MISSING VALUE AT RANDOM

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Abstract

There are several reasons why the data may be missing. They may be missing because equipment malfunctioned, the weather was terrible, or people got sick, or the data were not entered correctly. Missing data are problematic because most statistical procedures require a value for each variable. When a data set is incomplete, the data analyst has to decide how to deal with it. The most common decision is to use complete case analysis (also called list wise deletion) analyses cases with complete data. In this paper we use a two factor experiment to compare different methods of treatment of missing data to determine the best method among Expect Maximization algorithm, Multiple Imputation and Complete Case analysis when data is missing with varying percentages of missing data .and also to determine the accuracy the results shows that that the under the multiple imputation MCMC method is the best for the treatment of missing data for analysis of variance. Complete case analysis showed lower variation values thou is not the best method for treating missing data as it can substantially lower the sample size, leading to a severe lack of power and does not properly reflect statistical uncertainty.

Keywords:Missing data , Experimental design , Data Analysis

14 TOP: Topology & Analysis

STRICT EXTENSIONS OF FRAMES AND SPACES

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Abstract

Extensions of spaces have been constructed and used since the 19th century, for example, to form the complex sphere from the complex plane by adding a point at infinity. Once topological spaces were invented in the 20th century, completions and compactifications became important examples of extensions. Bernhard Banaschewski wrote in [1] that extension problems have a “philosophical charm” in that they seem to ask the question: “What possibilities in the unknown are determined by the known?” He went on to study all the extensions of a given space, and showed that the coarsest possible topology on an extension space gives the so-called *strict extension*.

In this talk, we discuss strict extensions in the pointfree setting: here, a strict extension is an onto frame homomorphism whose right adjoint generates the domain. This definition really generalises the definition for spaces, and provides insight into a classical result. In the pointfree setting, compactifications and completions are always strict extensions, and can be constructed using filters, just as the Hausdorff completion of a uniform space is constructed using minimal Cauchy filters.

This work has been submitted towards an M.Sc.

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Morrey Type Banach Spaces and Maximal Operator

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Abstract

Let $(\mathcal{F}, \|\cdot\|_{\mathcal{F}})$ be a Banach space of complex-valued measurable functions on \mathbb{R}_+^{n+1} . In this paper, we consider the Morrey-type Banach space $\mathcal{M}_{\mathcal{F}}(p, \lambda)$ as well as its weak type $\mathcal{M}_{\mathcal{F}}^*(1, \lambda)$. We develop the theory of Maximal operator and Fourier multipliers on these spaces. This research is a joint work with Vagif S. Guliyev and Ying Xiao.

Keywords: Morrey-type space, Hardy-Littlewood maximal function, Fourier multiplier

Symmetries in lightlike hypersurfaces of indefinite Kenmotsu manifolds

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Abstract

In this presentation, we deal with the symmetric property of lightlike hypersurfaces of indefinite Kenmotsu manifolds tangent to the vector field. We prove that there exist no weakly Ricci η -Einstein (or screen locally conformal) lightlike hypersurfaces in indefinite Kenmotsu manifolds, tangent to the structure vector field, if $\alpha + \beta + \gamma$ is nowhere zero, where α , β and γ are 1-form defined on the submanifold. Under a certain condition, we show that on weakly Ricci η -Einstein lightlike hypersurfaces in indefinite Kenmotsu manifolds, tangent to the structure vector field, the 1-form α vanishes and $\beta + \gamma = 0$. Moreover, a Ricci tensor for a special weakly lightlike hypersurfaces in indefinite Kenmotsu manifolds, tangent to the structure vector field, is parallel. We also prove that the geometry of the special weakly lightlike hypersurfaces is closely related to that of geodesibility and umbilicality of its tangent space and screen distribution, respectively. Under some conditions, a special weakly Ricci symmetric screen locally (or globally) conformal (or η -Einstein or Einstein) lightlike hypersurface of an indefinite Kenmotsu space form, tangent to the structure vector field, is locally symmetric, semi-symmetric and Ricci semi-symmetric.

Keywords: Indefinite Kenmotsu manifold; η -Einstein; Weakly Ricci symmetric lightlike hypersurfaces; Special weakly Ricci symmetric lightlike hypersurfaces; Totally geodesic; locally symmetric, semi-symmetric, Ricci semi-symmetric.

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GENERALIZED DOMINATION IN ORDERED BANACH ALGEBRAS

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Abstract

The following general problem in an *ordered Banach algebra* (OBA) has been investigated by various authors: If a and b are positive elements in an OBA such that $0 \leq a \leq b$ and if b has a certain property, under what conditions does a inherit that property? In the literature, this problem is referred to as the “domination problem”. In this talk we will introduce absolute value $|\cdot|$ in an OBA and obtain results for the domination problem under the more general inequality $|a| \leq |b|$. We will show that these results are applicable to positive operators on a Banach lattice. Furthermore, it will be demonstrated that some known results for the domination problem in OBAs continue to hold true if $0 \leq a \leq b$ is replaced by $|a| \leq |b|$.

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HARMONIC STRUCTURES and HARMONIC DENSITY

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Abstract

For a set of 4 real numbers $x_1 < x_2 < x_3 < x_4$ we define its cross ration as

$$k = \frac{(x_4 - x_1)(x_3 - x_2)}{(x_4 - x_3)(x_2 - x_1)}$$

and we say that the set is k -harmonic. In case of 1-harmonic sets we just speak of a harmonic set.

In this contribution we analyze the following question: For a fixed number $n \geq 5$ if real numbers are chosen arbitrarily, what is the largest number of k -harmonic 4-subsets that an n -set may have? For small n we obtain estimates for this maximal number of k -harmonic sets.

Here is the simplest interesting case: We find that for $n = 5$ at most 2 out of the five 4-subsets can be harmonic, but for the value $k = \frac{\sqrt{5}-1}{2}$ there can be 3 such k -harmonic subsets.

From this example it is interesting to draw up for a fixed k some tables of numbers of k -harmonic subsets.

The first table is that for $k = 1$. Then we search for other numbers $k \neq 1$ where the table may have larger entries than the first table in the respective position for $k = 1$. This may happen for some (but not many) algebraic numbers k like $k = \frac{\sqrt{5}-1}{2}$ or $k = \sqrt{2} - 1$ and some others.

We may formalize this by introducing the concept of a *harmonic structure*, and by defining *harmonic density*.

Our methods are computational geometry for the lower bounds, and the study of a particular class of partially ordered sets for the upper bounds.

This problem for $k = 1$ was first mentioned in a remark by A. Oldsknow [1].

Keywords: Harmonic set; harmonic density; golden section.

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Lebesgue-Nikodym Theorem for Vector Valued Additive Set Functions

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Abstract

We introduce a suitable definition of the integral that will allow us to extend to the vector valued case, the Lebesgue-Nikodym Theorem about the existence of a derivative of a finitely additive set function. We also present a further extension of the Fundamental Theorem of Calculus to additive set functions with values in an infinite dimensional normed space.

Brief Presentation of Perverse Sheaves as a Subcategory of the Derived Category of Complexes of Sheaves

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Abstract

Perverse sheaves are fundamental mathematical objects at the crossroads of algebraic geometry, topology, analysis and differential equations.

Let X be a stratified topological space with even dimensional strata. Its intended to present the definition and a brief description of the construction of perverse sheaves, $P(X)$, as the full subcategory of the bounded derived category of complexes of sheaves with cohomology constructible, $D^b(X)$, for the given stratification.

In this sense its going to be remarked the fact that the category of the perverse sheaves is an abelian category whose simple objects are the intersection homology sheaves of the strata of X .

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