



THREE DIMENSIONAL NON-HOMOGENEOUS THERMOELASTIC PROBLEM IN A THICK RECTANGULAR PLATE DUE TO INTERNAL HEAT GENERATION

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ABSTRACT. This paper is concerned with the determination of displacement and thermal stresses in a thick rectangular plate due to internal heat generation within it. Initially the plate is at arbitrary temperature $f(x, y, z)$. For times $t > 0$ heat is generated within the plate at a rate of $g(x, y, z, t)$ Btu/hr ft³, while the boundary surfaces are kept at zero temperature. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of circular functions. The results for displacement and stresses have been computed numerically and illustrated graphically.

1. INTRODUCTION

Plates are widely used in engineering applications such as aeronautical, naval and structural fields. They are made of homogeneous, laminated or functionally graded materials in various shapes, sizes and thicknesses depending on their applications, depending on the plate thickness. In the classical plate theory, the first order and the higher order shear deformation plate theories, those are usually use to formulates the differential equation of plates.

Ootao et al. [1] have studied the theoretical analysis of a three dimensional transient thermal stress problem for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction from the inner and outer surfaces. Tanigawa et al. [2] have studied the theoretical analysis of two dimensional thermoplastic bending deformation of plate subjected to partially distributed heat supply. Tanigawa at al. [3] have discussed stress analysis of a rectangular plate and its thermal stress intensity factor for compressive field. Vihak et al. [4] have investigated the solution of the plane thermoelasticity problem for a rectangular domain. Adam et al. [5] have determined

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the thermoelastic vibration of a laminated rectangular plate subjected to a thermal shock. Takuya et al. [6] have studied thermal buckling analysis of an orthotropic nonhomogeneous rectangular plate due to uniform heat supply. Mohsen et al. [7] have studied the thermoelastic stress field in a functionally graded curved beam, where the elastic stiffness varies in the radial direction, is considered. Salve et al. [8] studied an inverse transient quasi-static thermal stresses problem in a thin rectangular plate. Ghadle et al. [9] solved the quasi-Static thermal Stresses in a thick rectangular plate subjected to constant heat supply on the extreme edges where as the initial edges are thermally insulated. Recently [10] Deshmukh et al. studied the thermal stresses in a simply supported plate with thermal bending moments and determined the temperature distribution function subjected to the arbitrary initial heat supply.

In this article, we analyzed the three dimensional non-homogeneous heat conduction problem in a thick rectangular plate and determined the expressions for displacement and thermal stresses due to internal heat generation. Initially the plate is at arbitrary temperature $f(x, y, z)$. For times $t > 0$ heat is generated within the plate at a rate of $g(x, y, z, t)$ Btu/hr ft³, while the boundary surfaces are kept at zero temperature. The governing heat conduction equation has been solved by using integral transform technique. The results are obtained in series form in terms of circular functions. The results for displacement and stresses have been computed numerically and illustrated graphically.

To the author's knowledge, no literature on three dimensional non-homogeneous thermoelastic problem in a thick rectangular plate due to internal heat generation has been published. The results presented here will be more useful in engineering problem particularly, in the determination of the state of strain in a rectangular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

2. FORMULATION OF THE PROBLEM

Consider a thick rectangular plate with length a , width b and thickness c occupying the space D : $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$, as shown in figure 1. Initially the rectangular plate is at arbitrary temperature $f(x, y, z)$. For time $t > 0$, heat is generated within the plate at a rate of $g(x, y, z, t)$ Btu/hr ft³, while the remaining boundaries are kept at zero temperature. Under these realistic prescribed conditions, the displacement and thermal stresses in a thick rectangular plate due to internal heat generation are required to be determined.

The temperature $T(x, y, z, t)$ of the thick rectangular plate satisfies the heat conduction equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{g(x, y, z, t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}, \quad (2.1)$$

subject to the conditions,

$$T = 0, \quad \text{at all boundary surfaces, for } t > 0 \quad (2.2)$$

$$T = f(x, y, z), \quad \text{at } t = 0, 0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq c \quad (2.3)$$

where k and α are thermal conductivity and thermal diffusivity of the material of the plate.

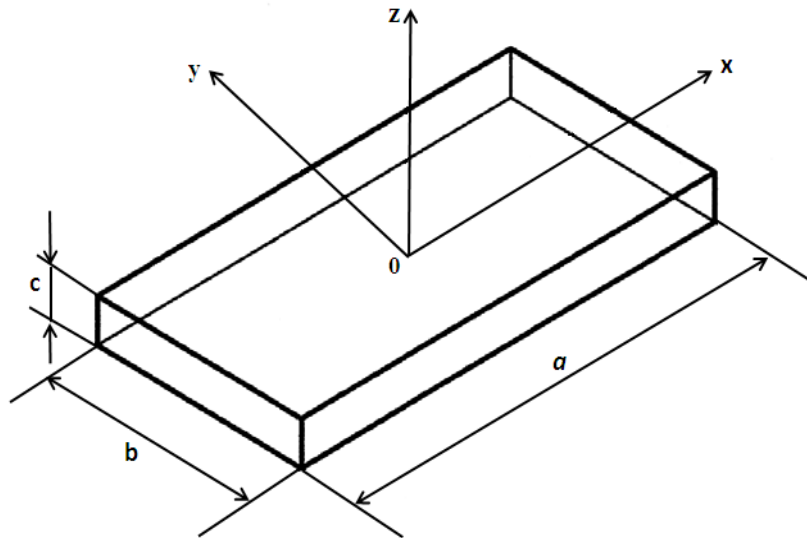


Figure 1. shows the geometry of the problem.

Here the plate is assumed sufficiently thick and considered free from traction. Since the plate is in a plane stress state without bending. Airy stress function method is applicable to the analytical development of the thermoelastic field. Airy stress function $U(x, y, z, t)$ which satisfy the following relation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)^2 U = -\lambda E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T \quad (2.4)$$

where λ and E are linear coefficient of the thermal expansion, Young's modulus elasticity of the material of the plate.

The displacement components u_x , u_y and u_z in the X , Y and Z direction are represented in the integral form as

$$u_x = \int \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial x^2} \right) + \lambda T \right] dx \quad (2.5)$$

$$u_y = \int \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} - \nu \frac{\partial^2 U}{\partial y^2} \right) + \lambda T \right] dy \quad (2.6)$$

$$u_z = \int \left[\frac{1}{E} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial z^2} \right) + \lambda T \right] dz \quad (2.7)$$

where ν is the poisson's ratio of the material of the plate.

The stress components in terms of U are given by

$$\sigma_{xx} = \left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) \quad (2.8)$$

$$\sigma_{yy} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial z^2} \right) \quad (2.9)$$

$$\sigma_{zz} = \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right) \quad (2.10)$$

Equations (2.1) to (2.10) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE HEAT CONDUCTION PROBLEM

To find the temperature function $T(x, y, z, t)$, we introduce the "triple-integral transform" and its corresponding "triple-inversion formula" as defined in [11] respectively as,

$$\begin{aligned} \bar{T}(\beta_m, \nu_n, \eta_p, t) &= \int_{x'=0}^a \int_{y'=0}^b \int_{z'=0}^c K(\beta_m, x') \cdot K(\nu_n, y') \cdot K(\eta_p, z') \\ &\quad \times T(x', y', z', t) \cdot dx' dy' dz' \end{aligned} \quad (3.1)$$

$$T(x, y, z, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} K(\beta_m, x) \cdot K(\nu_n, y) \cdot K(\eta_p, z) \cdot \bar{T}(\beta_m, \nu_n, \eta_p, t) \quad (3.2)$$

where the kernels

$$K(\beta_m, x) = \sqrt{\frac{2}{a}} \cdot \sin(\beta_m x) \quad (3.3)$$

$$K(\nu_n, y) = \sqrt{\frac{2}{b}} \cdot \sin(\nu_n y) \quad (3.4)$$

$$K(\eta_p, z) = \sqrt{\frac{2}{c}} \cdot \sin(\eta_p z) \quad (3.5)$$

and eigenvalues are

β_m is m^{th} root of transcendental equation $\sin(\beta_m \cdot a) = 0$

i.e.

$$\beta_m = \frac{m\pi}{a}, \quad m = 1, 2, 3, \dots \quad (3.6)$$

ν_n is n^{th} root of transcendental equation $\sin(\nu_n \cdot b) = 0$

i.e.

$$\nu_n = \frac{n\pi}{b}, \quad n = 1, 2, 3, \dots \quad (3.7)$$

η_p is p^{th} root of transcendental equation $\sin(\eta_p \cdot c) = 0$

i.e.

$$\eta_p = \frac{p\pi}{c}, \quad p = 1, 2, 3, \dots \quad (3.8)$$

On applying triple-integral transform defined in Eqs. (3.1) to Eqs. (2.1) – (2.3) and then using their inversions defined in Eqs. (3.2), one obtains the expressions of the temperature as

$$\begin{aligned} T(x, y, z, t) = & \frac{8}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t} \sin(\beta_m x) \sin(\nu_n y) \sin(\eta_p z) \\ & \times \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right] \end{aligned} \quad (3.9)$$

where

$$\bar{f}(\beta_m, \nu_n, \eta_p) = \int_{x'=0}^a \int_{y'=0}^b \int_{z'=0}^c \sin(\beta_m x') \sin(\nu_n y') \sin(\eta_p z') f(x', y', z') dx' dy' dz',$$

$$\bar{g}(\beta_m, \nu_n, \eta_p, t') = \int_{x'=0}^a \int_{y'=0}^b \int_{z'=0}^c \sin(\beta_m x') \sin(\nu_n y') \sin(\eta_p z') g(x', y', z', t') dx' dy' dz'.$$

Airy's Stress Function

Using equation (3.9) in (2.4), one obtains

$$U = \frac{8\alpha E}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \left[\frac{e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t}}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] \sin(\beta_m x) \sin(\nu_n y) \sin(\eta_p z) \quad (3.10)$$

$$\times \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right]$$

Displacement Components

Now using equations (3.9) and (3.10) in equations (2.5) to (2.7), one obtains the expressions for displacement as

$$u_x = \frac{-8\alpha}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t} \left[\frac{(\nu - 1)\beta_m^2 - 2(\nu_n^2 + \eta_p^2)}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] \left(\frac{\cos(\beta_m x)}{\beta_m} \right) \quad (3.11)$$

$$\times \sin(\nu_n y) \sin(\eta_p z) \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right]$$

$$u_y = \frac{-8\alpha}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t} \left[\frac{(\nu - 1)\nu_n^2 - 2(\beta_m^2 + \eta_p^2)}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] \sin(\beta_m x) \quad (3.12)$$

$$\times \left(\frac{\cos(\nu_n y)}{\nu_n} \right) \sin(\eta_p z) \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right]$$

$$u_z = \frac{-8\alpha}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t} \left[\frac{(\nu - 1)\eta_p^2 - 2(\beta_m^2 + \nu_n^2)}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] \sin(\beta_m x) \sin(\nu_n y) \quad (3.13)$$

$$\times \left(\frac{\cos(\eta_p z)}{\eta_p} \right) \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right]$$

Thermal Stresses

Now using equations (3.9) and (3.10) in equations (2.8) to (2.10), one obtains expressions for thermal stresses as

$$\begin{aligned} \sigma_{xx} = & \frac{-8\alpha E}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \left[\frac{e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t}}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] (\nu_n^2 + \eta_p^2) \sin(\beta_m x) \sin(\nu_n y) \sin(\eta_p z) \\ & \times \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right] \end{aligned} \quad (3.14)$$

$$\begin{aligned} \sigma_{yy} = & \frac{-8\alpha E}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \left[\frac{e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t}}{\beta_m^2 + \nu_n^2 + \eta_p^2} \right] (\beta_m^2 + \eta_p^2) \sin(\beta_m x) \sin(\nu_n y) \sin(\eta_p z) \\ & \times \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right] \end{aligned} \quad (3.15)$$

$$\begin{aligned} \sigma_{zz} = & \frac{-8\alpha E}{abc} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \left[\frac{e^{-k(\beta_m^2 + \nu_n^2 + \eta_p^2)t}}{\beta_m^2 + \nu_n^2} \right] (\nu_n^2 + \eta_p^2) \sin(\beta_m x) \sin(\nu_n y) \sin(\eta_p z) \\ & \times \left[\bar{f}(\beta_m, \nu_n, \eta_p) + \frac{\alpha}{k} \int_{t'=0}^t e^{k(\beta_m^2 + \nu_n^2 + \eta_p^2)t'} \bar{g}(\beta_m, \nu_n, \eta_p, t') dt' \right] \end{aligned} \quad (3.16)$$

4. NUMERICAL RESULTS

Setting

$$f(x, y, z) = e^t(a-x)(1-e^x)(b-y)(1-e^y)(c-z)(1-e^z),$$

$$g(x, y, z, t) = g_i \delta(x-x_1) \delta(y-y_1) \delta(z-z_1) \delta(t-\tau) \text{ Btu/hr.ft}^3$$

where δ is the Derac-delta function.

The heat source $g(x, y, z, t)$ is an instantaneous line heat source of strength $g_i=50$ Btu/hr.ft, situated at the center of the rectangular plate and releases its instantaneously at the time $t = \tau = 2$ hr.

Dimension

Length of rectangular plate $a = 3$ ft

Breadth of rectangular plate $b= 2$ ft

Height of rectangular plate $c= 1$ ft

Central length of rectangular plate $x_1 = 1.5$ ft

Central breadth of rectangular plate $y_1 = 1$ ft

central height of rectangular plate $z_1 = 0.5$ ft.

Material Properties

The numerical calculation has been carried out for an aluminum (Pure) rectangular plate with the material properties as,

Density $\rho = 169$ lb/ft³,

Specific heat = 0.208 Btu/lb⁰F,

Thermal conductivity $k = 117$ Btu/(hr.ft.⁰F),

Thermal diffusivity $\alpha = 3.33$ ft²/hr,

Poisson ratio $\nu = 0.35$,

Coefficient of linear thermal expansion $a_t = 12.84 \times 10^{-6}$ 1/F,

Lame constant $\mu = 26.67$,

Young's Modulus of elasticity $E = 70$ Gpa.

For convenience setting $A = \left(\frac{-8\alpha}{abc} \right)$ and $B = \left(\frac{-8\alpha E}{abc} \right)$

Considering

$$\lim_{m \rightarrow \infty} \beta_m = \lim_{n \rightarrow \infty} \nu_n = \lim_{p \rightarrow \infty} \eta_p = \infty$$

$$\lim_{m \rightarrow \infty} (e^{-k\beta_m^2.t}) = \lim_{n \rightarrow \infty} (e^{-k\nu_n^2.t}) = \lim_{p \rightarrow \infty} (e^{-k\eta_p^2.t}) = 0$$

Also the term $\cos(\beta_m x)$ and $\sin(\beta_m x)$ are bounded.

Thus necessary condition for convergence is satisfied, by applying D-Alemberts ratio test it can be easily verify that all the series in (3.11) to (3.16) are convergent. Also the term in the expression for displacements and stresses are negligible for large value of m, n and p it converges to zero at infinity.

The numerical calculation has been carried out with help of computational mathematical software Mathcad-2007, and the graphs are plotted with the help of Excel (MS Office-2007).

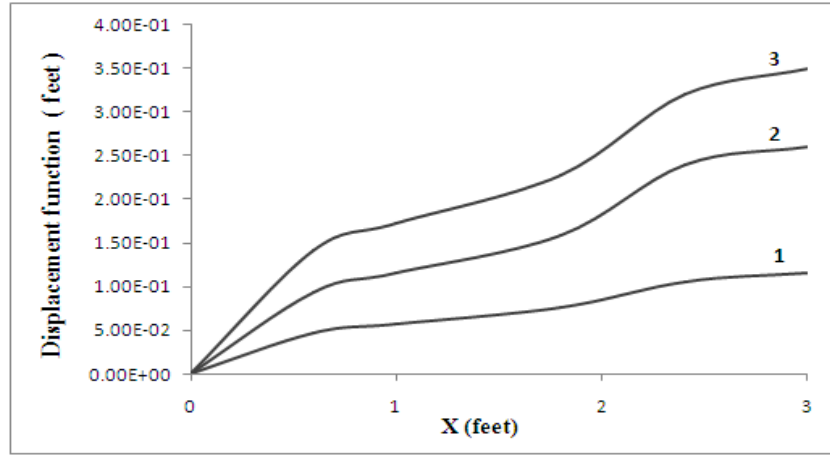


Figure. 2 The displacement function 1: $\frac{u_x}{A}$, 2: $\frac{u_y}{A}$ and 3: $\frac{u_z}{A}$ in X-direction.

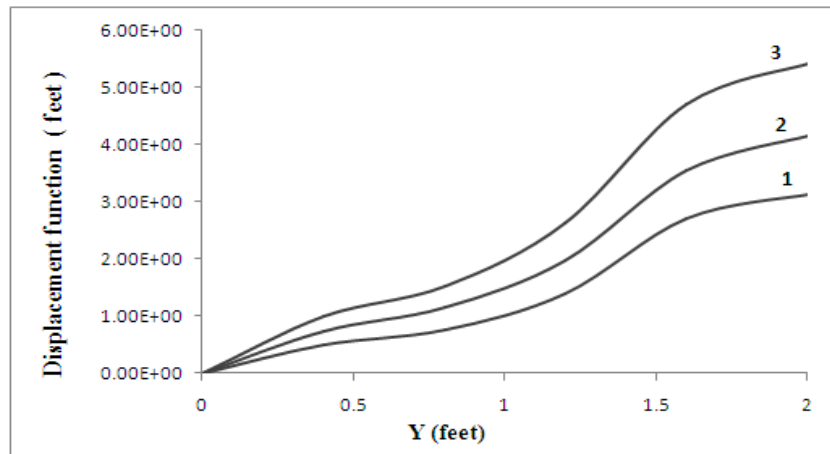


Figure. 3 The displacement function 1: $\frac{u_x}{A}$, 2: $\frac{u_y}{A}$ and 3: $\frac{u_z}{A}$ in Y-direction.

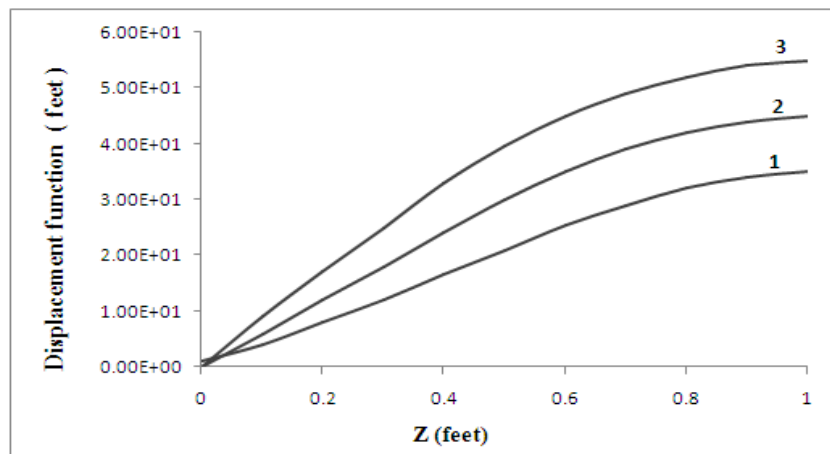


Figure. 4 The displacement function 1: $\frac{u_x}{A}$, 2: $\frac{u_y}{A}$ and 3: $\frac{u_z}{A}$ in Z-direction.

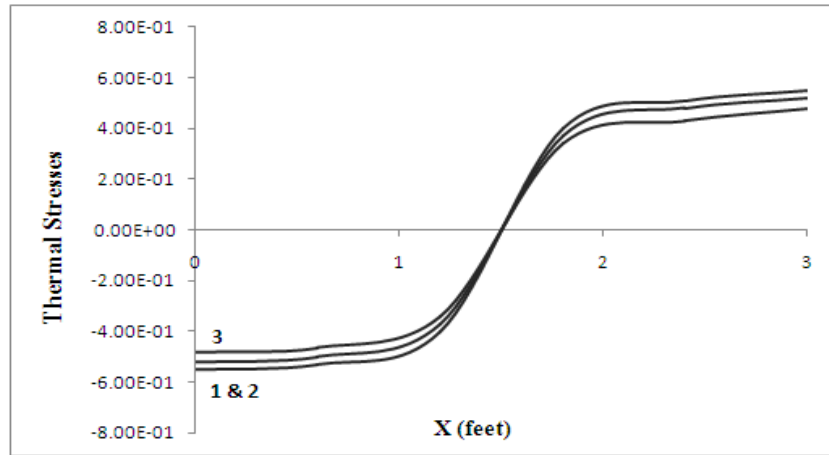


Figure. 5 The thermal stresses 1: $\frac{\sigma_{xx}}{B}$, 2: $\frac{\sigma_{yy}}{B}$ and 3: $\frac{\sigma_{zz}}{B}$ in X-direction.

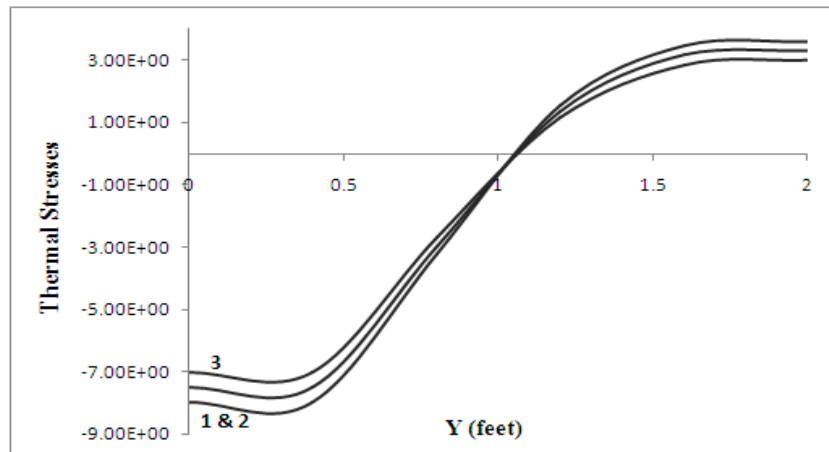


Figure. 6 The thermal stresses 1: $\frac{\sigma_{xx}}{B}$, 2: $\frac{\sigma_{yy}}{B}$ and 3: $\frac{\sigma_{zz}}{B}$ in Y-direction.

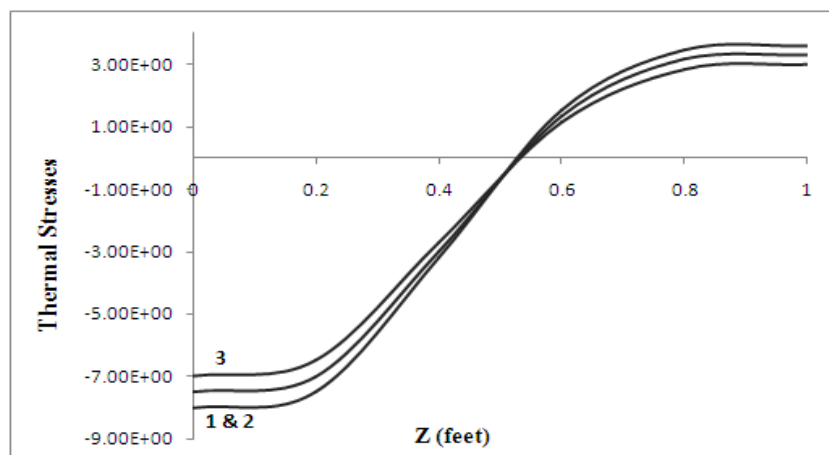


Figure. 7 The thermal stresses 1: $\frac{\sigma_{xx}}{B}$, 2: $\frac{\sigma_{yy}}{B}$ and 3: $\frac{\sigma_{zz}}{B}$ in Z-direction.

5. DISCUSSION

In this study, we analyzed the three dimensional non-homogeneous heat conduction problem in a thick rectangular plate. As an illustration, we carried out numerical calculations for a thick rectangular plate made up of aluminum (Pure). The heat source $g(x, y, z, t)$ is an instantaneous point heat source of strength g_i , is situated at the center of the rectangular plate in X, Y and Z direction and releases instantaneously at the time $t = \tau = 2$ hr. The thermoelastic behavior is examined such as displacement function and thermal stresses with the help of temperature and Airy's stress function.

From Figure 2, it is observed that the displacement function increases from initial edge towards the extreme edge and it develops the tensile stresses near the heated source in X direction. It is zero at the initial edge ($x = 0$).

From Figure 3, it is observed that the displacement function increases from initial edge towards the extreme edge and it develops the tensile stresses near the heated source in Y direction. It is zero at the initial edge ($y = 0$).

From Figure 4, it is observed that the displacement function increases from initial edge towards the extreme edge and it develops the tensile stresses near the heated source in Z direction. It is zero at the initial edge ($z = 0$).

From Figure 5, it is observed that the thermal stress increases from initial edge towards the extreme edge. It develops the compressive stresses within the region $0 \leq x \leq 1.5$ and tensile stresses within the region $1.5 \leq x \leq 3$ near the heated source in X direction and becoming zero at the center ($x = 1.5$).

From Figure 6, it is observed that the thermal stress increases from initial edge towards the extreme edge. It develops the compressive stresses within the region $0 \leq y \leq 1$ and tensile stresses within the region $1 \leq y \leq 2$ near the heated source in Y direction and becoming zero at the center ($y = 1$).

From Figure 7, it is observed that the thermal stress increases from initial edge towards the extreme edge. It develops the compressive stresses within the region $0 \leq z \leq 0.5$ and tensile stresses within the region $0.5 \leq z \leq 1$ near the heated source in Z direction and becoming zero at the center ($z = 0.5$).

6. CONCLUSION

In this article, we extend the problem studied by Ghadle et al. [9] without heat generation and study the three dimensional non-homogeneous thermoelastic problem in a thick rectangular plate due to internal heat generation.

We conclude that, the displacement function and thermal stress occurs near heat region, due to heat generation within the thick rectangular plate. From the figures of displacement, it can be observe that, displacement function develops tensile stresses in both X, Y and Z direction. Also from the figures of thermal stresses, we can be observe that, it develops compressive stresses as well as tensile stresses in both X, Y and Z direction.

The results, obtained here mainly applicable in engineering problems, particularly for industrial machines subjected to the heating such as the main shaft of a lathe, turbines, the roll of rolling mill and practical applications in air-craft structures. Also any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (3.9) and (3.11)-(3.16).

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