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AN INVERSE QUASI-STATIC THERMOELASTIC PROBLEM IN A THICK CIRCULAR PLATE

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Abstract. The aim of this work is to determine the unknown temperature, displacement and thermal stresses on the upper surface of a thick circular plate subjected to an arbitrary known interior temperature under a steady state field. The fixed circular edge is thermally insulated and the temperature of a lower surface plate is kept at zero. The governing heat conduction equation has been solved by using the Hankel transformation. The results are obtained in series form in terms of Bessels functions and results have been computed numerically and illustrated graphically.

1. INTRODUCTION

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their many applications in diverse fields. Firstly, the high velocity of modern aircrafts have given rise to aerodynamic heating which produces intense thermal stresses, reducing the strength of aircrafts structure. Secondly, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operation.

Ashida et al [1] studied the inverse transient thermoelastic problem for a composite circular disc constructed of transversely isotropic layers. Grysa et al [4] investigated an analytical approximate solution to an inverse problem of temperature field in the thermal stresses theory. Noda et al [6] discussed an inverse problem of coupled thermal stress fields in a circular cylinder. Roy Choudhary [7] studied a note of quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of a circle over the upper face with lower face

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at zero temperature and fixed circular edge thermally insulated. Deshmukh et al [3] solved an inverse transient problem of quasi-static thermal deflection of a thin clamped circular plate. Also Kulkarni et al [5] studied quasi-static thermal stresses in a thick circular plate. Recently Dange et al [2] studied two dimentional transient problems for a thick annular disc in thermoelasticity.

In this paper, an attempt is made to determine the unknown temperature, displacement and thermal stresses on the upper surface of a thick circular plate subjected to an arbitrary known interior temperature under a steady state field. The fixed circular edge is thermally insulated and the temperature of the lower half of the plate is kept at zero. The governing heat conduction equation has been solved by using the Hankel transform technique. The results are obtained in series form in terms of Bessels functions and results have been computed numerically and illustrated graphically.

In this work, we present some new interesting results of quasi-static thermal stresses in a thick circular plate. The above results were obtained under steady state field. The results presented here are useful in engineering problem particularly in the determination of the state of strain in a thick circular plate constituting foundations of containers for hot gases or liquids, in the foundations for furnaces etc.

2. FORMULATION OF THE PROBLEM

Consider a circular plate of radius a and thickness $2h$ occupying space D: $0 \leq r \leq a$, $-h \leq z \leq h$. Initially the plate is at zero temperature. Let the plate be subjected to an arbitrary known temperature $f(r)$ with in region $-h \leq z \leq h$. The lower surface $(z = -h)$ is zero and circular edges $(r = a)$ are thermally insulated. Assume that the boundary of the circular plate is free from traction. Under these more realistic prescribed conditions, the unknown temperature $g(r)$ which is at the upper of the plate and the quasi-static thermal stresses due to unknown temperature $g(r)$ need to be determined.

The differential equation governing the displacement potential function $\phi(r, z)$ is given in [8] as,

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = K\tau \tag{1}$$

where K is the restraint coefficient and the temperature change is given by $\tau = T - T_i$, where T_i is the initial temperature. The displacement function ϕ is known as Goodier's thermoelastic potential.

The steady-state temperature of the plate satisfies the heat condition equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{2}$$

subject to the boundary conditions,

$$\frac{\partial T}{\partial r} = 0 \qquad \text{at } r = a,\ -h \le z \le h \tag{3}$$

$$\frac{\partial T}{\partial z} + hs_1 T = f(r) \text{ (known)} \qquad \text{at } z = \xi,\ 0 \le r \le a \tag{4}$$

$$\frac{\partial T}{\partial z} - hs_2 T = 0 \qquad \text{at } z = -h,\ 0 \le r \le a \tag{5}$$

and

$$T = g(r) \text{ (unknown)} \qquad \text{at } z = h,\ 0 \le r \le a \tag{6}$$

where h_{s1} and h_{s2} are relative heat transfer coefficients on the upper and the lower surface of the thick circular plate.

The displacement function in the cylindrical coordinate system are represented by the Michell's function defined in [8] as,

$$u_r = \frac{\partial \phi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z} \tag{7}$$

$$u_z = \frac{\partial \phi}{\partial z} + 2(1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}. \tag{8}$$

The Michell's function M must satisfy

$$\nabla^2 \nabla^2 M = 0, \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}. \tag{10}$$

The components of the stresses are represented by the thermoelastic displacements potential ϕ and Michell's function M as

$$\sigma_{rr} = 2G\left[\frac{\partial^2\phi}{\partial r^2} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right] \tag{11}$$

$$\sigma_{\theta\theta} = 2G\left[\frac{1}{r}\frac{\partial\phi}{\partial r} - K\tau + \frac{\partial}{\partial z}\left(\nu\nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right] \tag{12}$$

$$\sigma_{zz} = 2G\left[\frac{\partial^2\phi}{\partial z^2} - K\tau + \frac{\partial}{\partial z}\left((2-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{13}$$

and

$$\sigma_{rz} = 2G\left[\frac{\partial^2\phi}{\partial r\partial z} + \frac{\partial}{\partial z}\left((1-\nu)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right] \tag{14}$$

where G and ν are the shear modulus and poisson's ratio respectively.

The boundary conditions on the traction free surfaces of the circular plate are

$$\begin{aligned} \sigma_{rr} = \sigma_{rz} = 0 \qquad & \text{at } r = a \\ \sigma_{zz} = \sigma_{rz} = 0 \qquad & \text{at } z = \pm h. \end{aligned} \tag{15}$$

Equations (1) to (15) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

To obtain the expressions for temperature $T(r,z)$ we introduce the finite Hankel transform over the variable r and its inverse transform defined in [10] as

$$\overline{T}(\lambda_n, z) = \int_0^a r J_o(\lambda_n r) T(r,z)\,dr \tag{16}$$

$$T(r,z) = \sum_{n=1}^{\infty}\left(\frac{2J_o(\lambda_n r)}{a^2 J_o^2(\lambda_n a)}\right)\overline{T}(\lambda_n, z) \tag{17}$$

where $\lambda_1, \lambda_2, \ldots$ are the roots of the transcendental equation

$$J_1(\lambda a) = 0 \tag{18}$$

with $J_n(x)$ is Bessel function of the first kind of order n.

This transform satisfies the relations

$$H\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]=-\lambda_n^2\overline{T}(\lambda_n,z) \tag{19}$$

and

$$H\left[\frac{\partial^2 T}{\partial z^2}\right]=\frac{d^2\overline{T}}{dz^2}. \tag{20}$$

On applying the finite Hankel transform defined in Eq.(16) to Eq.(2), one obtains

$$\frac{d^2\overline{T}}{dz^2}-\lambda_n^2\overline{T}=0 \tag{21}$$

where $\overline{T}$ is the Hankel transform of T.

On solving Eq. (21) under the conditions given in Eq. (4) and (5), one obtains

$$\overline{T}=\sum_{n=1}^{\infty}\overline{f}(\lambda_n)\left[\frac{\lambda n\cosh\left[\lambda_n(z+h)\right]+h_{s2}\sinh\left[\lambda_n(z+h)\right]}{(\lambda_n^2+h_{s1}h_{s2})\sinh\left[\lambda_n(\xi+h)\right]+\lambda_n(h_{s1}+h_{s2})\cosh\left[\lambda_n(\xi+h)\right]}\right]. \tag{22}$$

On applying the inverse Hankel transform defined in equation (17), one obtains the expression for the temperature as

$$\begin{aligned}T=&\left(\frac{2}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)J_o(\lambda_n r)}{J_o^2(\lambda_n a)}\right]\\&\times\left[\frac{\lambda n\cosh\left[\lambda_n(z+h)\right]+h_{s2}\sinh\left[\lambda_n(z+h)\right]}{(\lambda_n^2+h_{s1}h_{s2})\sinh\left[\lambda_n(\xi+h)\right]+\lambda_n(h_{s1}+h_{s2})\cosh\left[\lambda_n(\xi+h)\right]}\right]\end{aligned} \tag{23}$$

where $\overline{f}(\lambda_n)$ is the Hankel transform of $f(r)$.

$$\overline{f}(\lambda_n)=\int_0^a rJ_o(\lambda_n r)f(r)dr \tag{24}$$

UNKNOWN TEMPERATURE

The unknown temperature $g(r)$ can be obtained by substituting $z=h$ into Eq. (23) such that,

$$\begin{aligned}g(r)=&\left(\frac{2}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)J_o(\lambda_n r)}{J_o^2(\lambda_n a)}\right]\\&\times\left[\frac{\lambda n\cosh\left[2\lambda_n h\right]+h_{s2}\sinh\left[2\lambda_n h\right]}{(\lambda_n^2+h_{s1}h_{s2})\sinh\left[\lambda_n(\xi+h)\right]+\lambda_n(h_{s1}+h_{s2})\cosh\left[\lambda_n(\xi+h)\right]}\right]\end{aligned} \tag{25}$$

Since $T_i=0$, the temperature change is $\tau=T-T_i=T$.

MICHELL'S FUNCTION M

A suitable form of M satisfying Eq. (9) is given by

$$M = \left(\frac{2k}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}\lambda_n J_0(\lambda_n r)}{J_0^2(\lambda_n a)}\right] \times \{B_n[h_{s2}\cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]] \times C_n\lambda_n(z+h)[h_{s2}\sinh[\lambda_n(z+h)] + \lambda_n\cosh[\lambda_n(z+h)]]\}. \tag{26}$$

where B_n and C_n are arbitrary constants.

Assuming that the displacements function $\phi(r,z)$ has the form

$$\phi(r,z) = \sum_{n=1}^{\infty} D_n J_0(\lambda_n r) \times \left[\frac{(z+h)[h_{s2}\cosh[\lambda_n(z+h)] + \lambda_n\sinh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right] \tag{27}$$

and using ϕ in Eq. (1) one obtains

$$D_n = \frac{K\overline{f}(\lambda_n)}{a^2\lambda_n J_0^2(\lambda_n a)}. \tag{28}$$

Thus the equation (27) becomes

$$\phi(r,z) = \left(\frac{K}{a^2}\right)\sum_{n=1}^{\infty}\left\{\left[\frac{\overline{f}(\lambda_n)J_o(\lambda_n r)}{\lambda_n J_o^2(\lambda_n a)}\right] \times \left[\frac{(z+h)[h_{s2}\cosh[\lambda_n(z+h)] + \lambda_n\sinh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]\right\}. \tag{29}$$

DISPLACEMENT FUNCTIONS

Now using Eqs. (23)-(26) and (29) in (7) and (8), one obtains the expressions for displacement as

$$u_r = \left(\frac{K}{a^2}\right)\sum_{n=1}^{\infty}\left\{\left[\frac{\overline{f}(\lambda_n)J_1(\lambda_n r)}{J_o^2(\lambda_n a)}\right] \times \left[\frac{-(z+h)[h_{s2}\cosh[\lambda_n(z+h)] + \lambda_n\sinh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right] + B_n\lambda_n^2[h_{s2}\sinh[\lambda_n(z+h)] + \lambda_n\cosh[\lambda_n(z+h)]] + C_n\lambda_n^2\,[[h_{s2}\sinh[\lambda_n(z+h)] + \lambda_n\cosh[\lambda_n(z+h)]] + \lambda_n(z+h)\,[h_{s2}\cosh[\lambda_n(z+h)] + \lambda_n\sinh[\lambda_n(z+h)]]]\right\} \tag{30}$$

$$u_z = \left(\frac{K}{a^2}\right)\sum_{n=1}^{\infty}\left\{\left[\frac{\overline{f}(\lambda_n)J_0(\lambda_n r)}{J_o^2(\lambda_n a)}\right]\right.$$
$$\times\left[\frac{[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]]}{\lambda_n(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]$$
$$+\left[\frac{(z+h)[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]}{(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right] \tag{31}$$
$$-\ B_n\lambda_n^2[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]]$$
$$+\ C_n\lambda_n^2\left[2(1-2\nu)\left[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]\right]\right.$$
$$\left.\left.-\ \lambda_n(z+h)\left[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]\right]\right]\right\}.$$

QUASI-STATIC THERMAL STRESSES

Now using Eqs. (23)-(26) and (29) in (11)-(14), one obtains the expressions for stresses as follows

$$\sigma_{rr} = \left(\frac{2GK}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)J_0(\lambda_n r)}{J_o^2(\lambda_n a)}\right]\left\{\left(\lambda_n J_0(\lambda_n r)-\frac{J_1(\lambda_n r)}{r}\right)\right.$$
$$\times\left[\frac{-(z+h)[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]]}{(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]$$
$$-\ 2J_0(\lambda_n r)\left[\frac{[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]}{(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]$$
$$-\ B_n\lambda_n^2\left(\lambda_n J_0(\lambda_n r)-\frac{J_1(\lambda_n r)}{r}\right)[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]$$
$$+\ C_n\lambda_n^2\left[\ 2\nu\lambda_n J_0(\lambda_n r)[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]\right.$$
$$+\left(\lambda_n J_0(\lambda_n r)-\frac{J_1(\lambda_n r)}{r}\right)\times[[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]$$
$$\left.\left.+\ \lambda_n(z+h)\left[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]\right]\right]\right]\Big\} \tag{32}$$

$$\sigma_{\theta\theta} = \left(\frac{2GK}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)J_0(\lambda_n r)}{J_o^2(\lambda_n a)}\right]\left\{\left(-\frac{J_1(\lambda_n r)}{r}\right)\right.$$
$$\times\left[\frac{(z+h)[h_{s2}.\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]]}{(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]$$
$$-\ 2J_0(\lambda_n r)\left[\frac{(z+h)[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]}{(\lambda_n^2+h_{s1}h_{s2})\sinh[\lambda_n(\xi+h)]+\lambda_n(h_{s1}+h_{s2})\cosh[\lambda_n(\xi+h)]}\right]$$
$$+\ B_n\lambda_n^2\left(\frac{J_1(\lambda_n r)}{r}\right)[h_{s2}.\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]$$
$$+\ C_n\lambda_n^2\left[\ 2\nu\lambda_n J_0(\lambda_n r)[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]\right.$$
$$+\ \left(\frac{J_1(\lambda_n r)}{r}\right)\times[[h_{s2}\sinh[\lambda_n(z+h)]+\lambda_n\cosh[\lambda_n(z+h)]]$$
$$\left.\left.+\ \lambda_n(z+h)\left[h_{s2}\cosh[\lambda_n(z+h)]+\lambda_n\sinh[\lambda_n(z+h)]\right]\right]\right]\Big\} \tag{33}$$

$$
\begin{aligned}
\sigma_{zz} = & \left(\frac{2GK}{a^2}\right) \sum_{n=1}^{\infty} \left[\frac{\overline{f}(\lambda_n) J_0(\lambda_n r)}{J_o^2(\lambda_n a)}\right] \\
& \times \left\{ \left[\frac{\lambda_n (z+h)[h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]} \right] \right. \\
& - B_n \lambda_n^3 [h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]] \\
& + C_n \lambda_n^3 [\, (1-2\nu)\, [h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]] \\
& \left. - \lambda_n(z+h)\, [h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]]]\right\}
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
\sigma_{rz} = & \left(\frac{2GK}{a^2}\right) \sum_{n=1}^{\infty} \left[\frac{\overline{f}(\lambda_n) J_1(\lambda_n r)}{J_o^2(\lambda_n a)}\right] \\
& \times \left\{ \left[\frac{-[h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]} \right] \right. \\
& - \left[\frac{\lambda_n (z+h)[h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]} \right] \\
& + B_n \lambda_n^3 [h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]] \\
& + C_n \lambda_n^3 [\, 2\nu\, [h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]] \\
& \left. + \lambda_n(z+h)\, [h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]]]\right\}.
\end{aligned} \tag{35}
$$

Now in order to satisfy the boundary conditions given in the equation (15), we use equations (33)-(35) for B_n and C_n to obtain

$$
B_n = \frac{(1-2\nu)}{\lambda_n^3 \, [(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]]}
$$

and

$$
C_n = \frac{1}{\lambda_n^3 \, [(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]]}.
$$

Using these values of B_n and C_n in Eqs. (30) to (35) one obtain the expressions for displacement and stresses as follows

$$
\begin{aligned}
u_r = & \left(\frac{2K(1-2\nu)}{a^2}\right) \sum_{n=1}^{\infty} \left[\frac{\overline{f}(\lambda_n) J_1(\lambda_n r)}{J_o^2(\lambda_n a)}\right] \\
& \times \left[\frac{h_{s2} \sinh[\lambda_n(z+h)] + \lambda_n \cosh[\lambda_n(z+h)]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]} \right]
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
u_z = & \left(\frac{2K(1-2\nu)}{a^2}\right) \sum_{n=1}^{\infty} \left[\frac{\overline{f}(\lambda_n) J_0(\lambda_n r)}{J_o^2(\lambda_n a)}\right] \\
& \times \left[\frac{h_{s2} \cosh[\lambda_n(z+h)] + \lambda_n \sinh[\lambda_n(z+h)]}{(\lambda_n^2 + h_{s1}h_{s2}) \sinh[\lambda_n(\xi+h)] + \lambda_n(h_{s1}+h_{s2}) \cosh[\lambda_n(\xi+h)]} \right]
\end{aligned} \tag{37}
$$

$$\sigma_{rr} = \left(\frac{4KG(1-\nu)}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)J_1(\lambda_n r)}{rJ_o^2(\lambda_n a)}\right] \times \left[\frac{h_{s2}\sinh\left[\lambda_n(z+h)\right]+\lambda_n\cosh\left[\lambda_n(z+h)\right]}{(\lambda_n^2+h_{s1}h_{s2})\sinh\left[\lambda_n(\xi+h)\right]+\lambda_n(h_{s1}+h_{s2})\cosh\left[\lambda_n(\xi+h)\right]}\right] \tag{38}$$

$$\sigma_{\theta\theta} = \left(\frac{-4KG(1-\nu)}{a^2}\right)\sum_{n=1}^{\infty}\left[\frac{\overline{f}(\lambda_n)}{J_o^2(\lambda_n a)}\right]\left(\lambda_n J_0(\lambda_n r)-\frac{J_1(\lambda_n r)}{r}\right) \times \left[\frac{h_{s2}\sinh\left[\lambda_n(z+h)\right]+\lambda_n\cosh\left[\lambda_n(z+h)\right]}{(\lambda_n^2+h_{s1}h_{s2})\sinh\left[\lambda_n(\xi+h)\right]+\lambda_n(h_{s1}+h_{s2})\cosh\left[\lambda_n(\xi+h)\right]}\right] \tag{39}$$

$$\sigma_{zz} = 0 \tag{40}$$

and

$$\sigma_{rz} = 0. \tag{41}$$

SPECIAL CASE

Setting

$$f(r) = (r^2-a^2)^2 \tag{42}$$

and applying the Hankel transform as defined in Eqs. (24) to (42), one obtains

$$\overline{f}(\lambda_n) = \int_0^a r(r^2-a^2)^2 J_0(\lambda_n r)dr,$$

$$\overline{f}(\lambda_n) = \frac{8a\left\{(8-a^2\lambda_n^2)J_1(\lambda_n a)-4a\lambda_n J_0(\lambda_n a)\right\}}{\lambda_n^5}. \tag{43}$$

4. NUMERICAL CALCULATIONS

Numerical calculations have been carried out for a steel ($SN\ 50C$) plate with parameters chosen $a = 1m$, $h = 0.2m$. The thermal diffusivity is given by $k = 15.9 \times 10^6 (m^2 s^{-1})$ and the Poisson ratio by $\nu = 0.281$. The transcendental roots of $J_1(\lambda_n a)$ as in [9] are $\lambda_1 = 3.8317$, $\lambda_2 = 7.0156$, $\lambda_3 = 10.1735$, $\lambda_4 = 13.3237$, $\lambda_5 = 16.470$, $\lambda_6 = 19.6159$, $\lambda_7 = 22.7601$, $\lambda_8 = 25.9037$, $\lambda_9 = 29.0468$, $\lambda_{10} = 32.18$. The relative heat transfer coefficients h_{s1}=10 and h_{s2}=5.

For convenience we set $\alpha = \left(\frac{-16}{10^2 a}\right)$, $\beta = \left(\frac{16K}{10^2 a}\right)$ and $\gamma = \left(\frac{32GK}{10^2 a}\right)$ in Eqs. (25), (36) to (39). The numerical expressions for the unknown temperature, displacement and stress function are obtained in Eqs. (25), (36) to (39).

In order to examine the influence of an unknown temperature on the upper surface of the circular plate, we performed numerical calculations for $z = \frac{h}{2}$, $r = 0, 0.2, 0.4, 0.6, 0.8, 1m$ and $\xi = -0.2, -0.1, 0, 0.1, 0.2m$. Numerical variations in radial directions are shown in the figures with help of a computer programme.

5. CONCLUDING REMARKS

In this paper we have considered a thick circular plate which is free from traction subjected to an arbitrary known interior temperature. Expressions are determined for an unknown temperature and the consequent displacement and stress function. As a special case a mathematical model is constructed for

$$f(r) = (r^2 - a^2)^2$$

and numerical calculations were performed. The thermoelastic behaviour such as an unknown temperature, displacement and stresses with the help of arbitrary known interior temperature is examined.

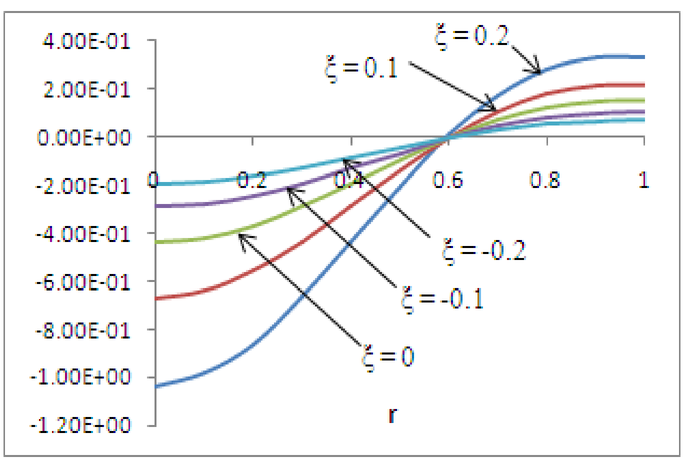

Figure. 1 Unknown temperature $g(r)/\alpha$ in radial direction.

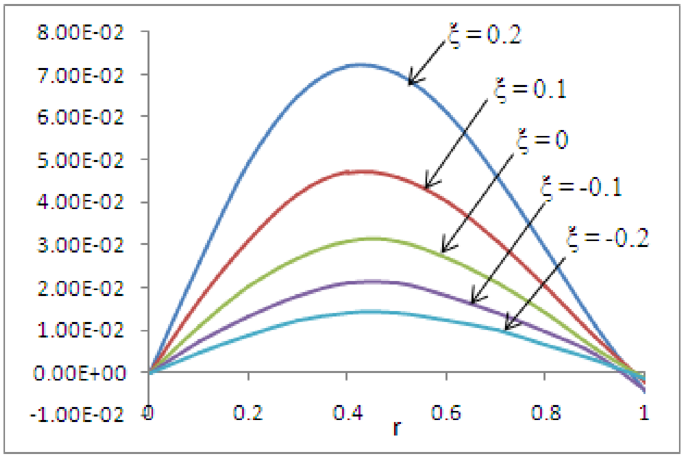

Figure. 2 Radial displacement function u_r/β in radial direction.

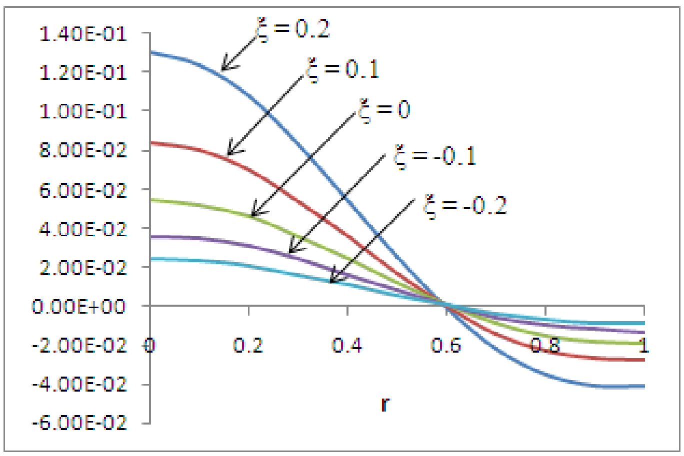

Figure. 3 Axial displacement function u_z/β in radial direction.

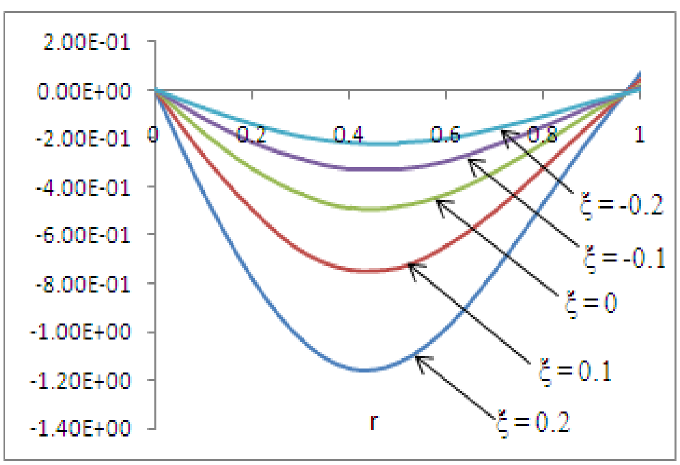

Figure. 4 Radial stress function σ_{rr}/γ in radial direction.

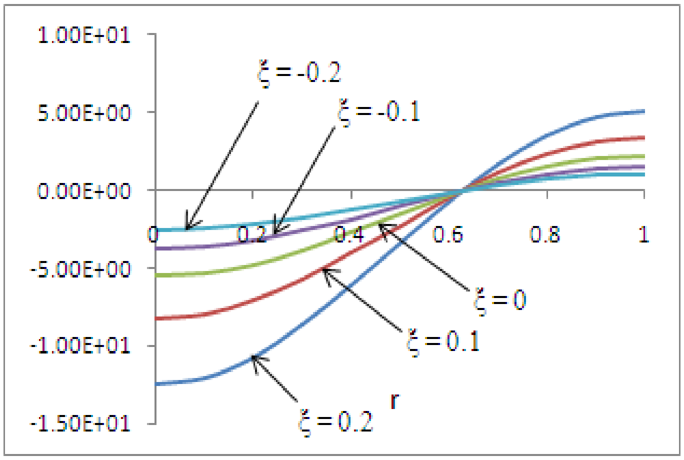

Figure. 5 Angular stress function $\sigma_{\theta\theta}/\gamma$ in radial direction.

Figure 1, the unknown temperature $g(r)$ develops tensile stress within annular region $0.6 \leq r \leq 1$ and compressive stress in circular region $0 \leq r \leq 0.6$ in the radial direction.

From figure 2, the radial displacement u_r shows the normal curve and it is zero at $r = 0$, the circular boundary of the circular plate. Also it develops the tensile stress in the radial direction.

From figure 3, the axial displacement u_z develops tensile stress within circular region $0 \leq r \leq 0.6$ and compressive stress in annular region $0.6 \leq r \leq 1$.

From figure 4, the radial stress σ_{rr} shows the normal curve and it is zero at $r = 0$, the circular boundary of the circular plate. Also it develops the compressive stresses in the radial direction.

From figure 5, the angular stress $\sigma_{\theta\theta}$ develops tensile stress within annular region $0.7 \leq r \leq 1$ and compressive stress in circular region $0 \leq r \leq 0.7$ in the radial direction.

Thus we are able to find the displacement and stress components which occur near heat source. With an increases in temperature the circular plate will tend to expand in the radial direction as well as in the axial direction. In the plane state of stress the stress components σ_{zz} and σ_{rz} are zero. Also from the Figures 2 and

3, it can be observed that the displacement occurs around the center towards in the downward direction. So it may be concluded that due unknown temperature the circular plate expands in the axial direction and bends concavely at the center. This expansion is inversely proportional to the thickness of the circular plate.

The results obtained here are more useful in engineering problems particularly in the determination of the state of strain in a thick circular plate. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expressions (25), (36) to (39).

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